

## Math 5330 Test I

Name \_\_\_\_\_

1. If

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

find a lower triangular matrix  $L$  and an upper triangular matrix  $U$  such that  $A = LU$ .

2. Find the condition number of

$$A = \begin{bmatrix} 1.00001 & 1 \\ & 1 & 1 \end{bmatrix}$$

If the machine precision is  $\epsilon_{mach} = 10^{-15}$ , about how many significant decimal digits would you expect in the solution of  $Ax = b$ , if Gaussian elimination is done?

3. Approximately how many multiplications does the following MATLAB program do, for large  $N$ :

```
X = 1;
for I=1:N
    for J=1:I
        for K=1:J
            X = X*X;
        end
    end
end
```

4. Define:

- a. orthogonal matrix
- b. upper Hessenberg matrix
- c. positive definite matrix
- d.  $\|x\|_p$ , if  $x$  is a vector and  $1 \leq p < \infty$
- e.  $\|A\|_p$ , if  $A$  is a matrix

5. An  $N$  by  $N$  band matrix has  $\sqrt{N}$  non-zero diagonals below the main diagonal and the same number above. If  $N$  is large, approximately how many multiplications are done:
  - a. during the forward elimination, if no pivoting is done?
  - b. during the forward elimination, if partial pivoting is done?
  - c. during back substitution, if no pivoting is done?
  - d. during back substitution, if partial pivoting is done?
  
6. a. If a matrix is decomposed into its (strictly) lower triangular, diagonal, and (strictly) upper triangular parts,  $A = L + D + U$ , the Jacobi iterative method for solving  $Ax = b$  will converge if and only if all eigenvalues of what matrix are less than 1 in absolute value?
  - b. Same question, for the Gauss-Seidel method.
  
  - c. Using parts [a.] and [b.], show that both Jacobi and Gauss-Seidel methods will converge if  $A$  is either upper triangular or lower triangular, and all its diagonal elements are nonzero. (Hint: the eigenvalues of an upper or lower triangular matrix are its diagonal entries.)