Math 5329, Final

Name ____________________________

1. a. Let $T_3(x)$ be the Taylor polynomial of degree 3 which matches $f(x), f'(x), f''(x)$ and $f'''(x)$ at $x = 0$, where $f(x) = sin(0.1\ x)$. Find the best possible bound on

   $max_{-0.5 \le x \le 0.5} |T_3(x) - f(x)| \le$

   b. Let $L_3(x)$ be the Lagrange polynomial of degree 3 which matches $f(x)$ at $x = -2, -0.5, 0.5$ and 2, where $f(x) = sin(0.1\ x)$. Find the best possible bound on

   $max_{-0.5 \le x \le 0.5} |L_3(x) - f(x)| \le$

2. a. A root finder gives consecutive errors of $e_8 = 10^{-3}, e_9 = 10^{-5}, e_{10} = 10^{-11}$. Estimate the order of the method.

   b. A quadrature method gives an error of 10^{-5} when $h = 10^{-2}$ and 10^{-11} when $h = 10^{-4}$. Estimate the (global) order of the method.

   c. A differential equation solver gives an answer $u(1) = 0.148888$ when $h = 0.1$, and $u(1) = 0.140666$ when $h = 0.01$, and $u(1) = 0.140600$ when $h = 0.001$. Estimate the (global) order of the method.

3. Consider the system

$$f_1(x_1, ..., x_N) = a_{11}x_1 + ... + a_{1N}x_N - b_1 = 0$$

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$$f_N(x_1, ..., x_N) = a_{N1}x_1 + ... + a_{NN}x_N - b_N = 0$$

a. Write out Newton's method for solving this system, and show that it will converge in a single iteration (regardless of starting solution) to the solution of $Ax = b$, that is, to $A^{-1}b$.

b. Modify Newton's method by setting all off-diagonal terms of the Jacobian matrix equal to zero, and show that it is now equivalent to the Jacobi iteration for solving $Ax = b$.

4. a. Is the following method stable? (Justify answer)

$$\frac{U_{k+1}+4U_k-5U_{k-1}}{6h} = \frac{2}{3}f(t_k, U_k) + \frac{1}{3}f(t_{k-1}, U_{k-1})$$

b. Is the method consistent (with $u' = f(t, u)$)? (Justify answer)

c. Will the finite difference solution converge to the differential equation solution as $h \to 0$?

5. a. Will the iteration $x_{n+1} = 2 + (x_n - 2)^4$ converge when x_0 is sufficiently close to the root $r = 2$? If so, what is the order of convergence? (Justify your answer theoretically)

b. What is the range of values of x_0 for which this iteration will converge to $r = 2$?

c. There is a second root, that is, another value such that if $x_n = r$, $x_{n+1} = r$ also. Find this root, and the range of values of x_0 for which the iteration will converge to this root.