Math 5329, Test III

Name _____

1. Find A, B, C which make the quadrature formula below as high order as possible:

   $\int_0^h f(x)dx \approx Ahf(0.2h) + Bhf(0.5h) + Chf(0.8h)$

2. Use Taylor series to find the error in the approximation:

   $u'(t) \approx \frac{-u(t+2h)+8u(t+h)-8u(t-h)+u(t-2h)}{12h}$

3. If the third order Taylor series method (two more terms than Euler's method) is used to solve $u' = t^2u^3$, write u_{n+1} in terms of h, t_n and u_n only. $(t_n = nh, u_n \approx u(t_n))$

4. Write the third order equation:

   $u''' - cos(u'') + e^t u' + 4t\ sin(u) = 25$

   as a system of three first order equations, of the form:

   $u_1' = f_1(t, u_1, u_2, u_3) =$
   $u_2' = f_2(t, u_1, u_2, u_3) =$
   $u_3' = f_3(t, u_1, u_2, u_3) =$

5. a. Is the method $\frac{3}{2}U_{n+1} - 2U_n + \frac{1}{2}U_{n-1} = hf(t_{n+1}, U_{n+1})$ (for approximating $u' = f(t, u)$) stable?

b. Is it explicit or implicit?

c. Calculate the truncation error. (Hint: put in normalized form first.)