

### Math 5329, Test III

Name \_\_\_\_\_

1. Determine the (global) order of the quadrature rule:

$$\int_a^b f(x)dx \approx \sum_{i=1}^N \left[ \frac{h}{4} f(x_{i-1}) + \frac{3h}{4} f\left(x_{i-1} + \frac{2h}{3}\right) \right]$$

2. Find  $A, B, C$  such that the approximation  $u'(t) \approx \frac{Au(t)+Bu(t-h)+Cu(t-2h)}{h}$  is as high order as possible.

3. If the second order Taylor series method (one more term than Euler's method) is used to solve  $u' = t^2\sqrt{1+u^2}$ , write  $u_{n+1}$  in terms of  $h, t_n$  and  $u_n$  only. ( $t_n = nh, u_n \approx u(t_n)$ )

4. Reduce

$$\begin{aligned}y'' &= 3y'y - e^tz \\z'' &= z'z - \sqrt{y}\end{aligned}$$

to a system of 4 first order equations.

5. a. Is the method  $11U_{n+1} - 18U_n + 9U_{n-1} - 2U_{n-2} = 6hf(t_{n+1}, U_{n+1})$   
(for approximating  $u' = f(t, u)$ ) stable?
- b. Is it explicit or implicit?
- c. Is it consistent? (Extra credit: what is the truncation error?)