Math 5329, Test II

Name ______________________________

1. If

$$A = \begin{bmatrix} 1 & a \\ a & 2 \end{bmatrix}$$

Find the range of values of a for which the Gauss-Seidel method will converge, when applied to a system with matrix A.

2. a. Find the LU decomposition (without pivoting) of

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & \frac{61}{16} \end{bmatrix}$$

b. Now find the Cholesky decomposition LL^T of A (not necessarily the same L as part (a)).

c. Prove that A is positive definite. (Hint: Use part (b))

3. Do several iterations of the inverse power method to find the smallest eigenvalue (in absolute value) of A, and the corresponding eigenvector, if

$$A = \begin{bmatrix} -4/6 & 2/6 \\ 5/6 & -1/6 \end{bmatrix}$$

4. If $p_3(x)$ is the polynomial of degree 3 which interpolates $f(x) = ln(x)$ at $x = 1.0, 1.1, 1.2, 1.3$, find as small a bound as possible on $max_{1.1 \leq x \leq 1.2} |p_3(x) - f(x)|$. (Note: the range is only $(1.1, 1.2)$).

5. Set up (don't try to solve) the equations to determine the cubic spline

$$\begin{aligned} s(x) &= a + bx + cx^2 + dx^3 \quad -1 \leq x \leq 0 \\ & \quad\; e + fx + gx^2 + hx^3 \quad 0 \leq x \leq 1 \end{aligned}$$

which interpolates to $f(x) = sin(\frac{\pi}{2}x)$ at $x = -1, 0, 1$ and which matches $f''(x)$ at the endpoints $x = -1$ and $x = 1$.