

Math 5329, Test II

Name _____

1. a. Find the LU decomposition (no pivoting necessary) for

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

- b. Solve $Ax = b$, where $b = (7, 1, 3)$ by first solving $Ly = b$, then $Ux = y$.

2. If $A = L + D + U$ ($L =$ strictly lower triangular, U is strictly upper triangular and D is diagonal), what is the matrix whose eigenvalue must be less than one in absolute value for convergence, for the

- a. Jacobi Iterative Method
- b. Gauss-Seidel Iterative Method
- c. SOR Iterative Method

3. Do several iterations of the inverse power method to find the smallest eigenvalue (in absolute value) of A , and the corresponding eigenvector, if

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

4. If $p_N(x)$ is the polynomial of degree N which interpolates $f(x) = \cos(3x)$ at $N + 1$ uniformly spaced points between 0 and π , find a bound, involving only N , on $\max(0 \leq x \leq \pi) |p_N(x) - f(x)|$. Will your bound go to zero as $N \rightarrow \infty$?

5. Determine the equations which must be satisfied for

$$\begin{aligned} s(x) &= a(x-2)^2 + b(x-1)^3 & x \leq 1 \\ & c(x-2)^2 & 1 \leq x \leq 3 \\ & d(x-2)^2 + e(x-3)^3 & 3 \leq x \end{aligned}$$

to be a cubic spline.