Math 5329, Test I

Name ______________________________

1. a. Find $T_n(x)$, the Taylor series of degree n for the function $f(x) = cosh(x)$, expanded around $a = 0$. Assume n is even.
(Hint: $\frac{d}{dx}cosh(x) = sinh(x), \frac{d}{dx}sinh(x) = cosh(x)$, sinh(0)=0, cosh(0)=1)

b. Find $E_n(x)$, the error in $T_n(x)$, and find a reasonable upper bound on $E_n(10)$. You can use the fact that $sinh(x)$ is a monotone increasing function.

c. Estimate the number of terms n required for $T_n(10)$ to approximate $cosh(10)$ to an accuracy of 10^{-4}.

d. Would you expect roundoff error to be a serious concern in computing $T_n(10)$ in part (c)? Why or why not?

2. a. To find a maximum or minimum of a function F(x,y), in calculus we set both partial derivatives to 0 and solve the resulting system of two equations. Explicitly write out what Newton's method looks like when applied to this system, in terms of F and its derivatives.

b. If F(x,y) is a quadratic polynomial ($F(x, y) = a + bx + cy + dx^2 + exy + fy^2$), what can you say about convergence of Newton's method?

3. It can be shown that for Mueller's method, $e_{n+1} \approx M e_n e_{n-1} e_{n-2}$. If Mueller's method is order α, ie, $e_{n+1} \approx C e_n^\alpha$, find an equation satisfied by α. Then use any method we have studied to find a root of this equation. (Hint: First write e_{n-1} and e_{n-2} in terms of e_n.)

4. About how many bisection iterations should be required to obtain an error less than ϵ, knowing that f(a) and f(b) have opposite signs?

5. Estimate the order of convergence for:

   a. Newton's method applied to $f(x) = (x-3)^3(x-4)$, starting near the root r=3.

   b. Same as (a) but starting near the root r=4.

   c. Same as (a) but using Secant method.

   d. Same as (a) but using Secant method and starting near the root r=4.

   e. A root finder which produces consecutive errors of $10^{-5}, 10^{-7}$ and 10^{-12}.

   f. The iteration $x_{n+1} = g(x_n)$ if $r = g(r)$ and $g'(r) = g''(r) = 0$ but $g'''(r) \neq 0$, and you start near the root r.

   g. The bisection method.

   h. The method $x_{n+1} = x_n - f(x_n)/f'(x_0)$.