Math 4329, Final

Name ______________________________

1. Use the power method to find the largest (in absolute value) eigenvalue of

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 8 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Start with $(1, 5, 1)$ and do 3 iterations. What is the corresponding eigenvector?

2. a. If $p_4(x)$ is the fourth degree Lagrange polynomial which satisfies $p_4(x_i) = f(x_i),\ i = 0, 1, 2, 3, 4$, give a formula for the error $f(x) - p_4(x)$ at an arbitrary point x.

b. If $T_4(x)$ is the fourth degree Taylor polynomial which satisfies $T_4(a) = f(a), T_4'(a) = f'(a), T_4''(a) = f''(a), T_4'''(a) = f'''(a), T_4^{iv}(a) = f^{iv}(a)$, give a formula for the error $f(x) - T_4(x)$ at an arbitrary point x.

3. If the second order Taylor series method (one more term than Euler's method) is used to solve $u' = -tu^3$, write u_{n+1} in terms of h, t_n and u_n. ($t_n = nh, u_n \approx u(t_n)$)

4. Do **one** iteration of Newton's method, starting from $(0,0)$, to solve:
$f(x,y) = \sqrt{x+1} + xy + 3 = 0$
$g(x,y) = sin(x+2y) - ln(1+x) = 0$

5. How should A, r be chosen to make the approximation:

$\int_{-1}^{1} f(x)dx \approx Af(-r) + Af(0) + Af(r)$

as high degree of precision as possible? What is the degree of precision then?

6. Consider the linear system:

$$\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$$

a. Write out the equations for the Jacobi iterative method for solving this system (don't actually do any iterations).

b. Write out the equations for the Gauss-Seidel iterative method for solving this system.

c. True or False: the Jacobi iterative method (6a) will converge for *any* starting vector (x_0, y_0, z_0). Give a reason for your answer.

d. Given that

$$A^{-1} = \begin{bmatrix} 0.1419 & -0.0878 & 0.0068 \\ 0.0135 & 0.2297 & -0.0946 \\ -0.0203 & 0.1554 & 0.1419 \end{bmatrix}$$

find the condition number of A (using L_∞ norm). If you were to solve the linear system above using Gaussian elimination with partial pivoting, would you expect serious roundoff errors?

7. Will the iteration $x_{n+1} = \frac{3}{4}x_n + 1/x_n^3$ converge when x_0 is sufficiently close to the root $r = \sqrt{2}$? (Justify your answer theoretically, without actually iterating the formula.) If it converges, give the order of convergence.

8. a. Write the second order differential equation $u'' - 5u' - u = sin(t)$ as a system of two first order equations, that is, in the form:

$u' = f(t, u, v) =$

$v' = g(t, u, v) =$

b. Now write out the formulas for u_{n+1}, v_{n+1} for Euler's method applied to this system of first order equations:

$u_{n+1} =$

$v_{n+1} =$

9. a. A root finder gives consecutive errors of $e_8 = 10^{-3}, e_9 = 10^{-5}, e_{10} = 10^{-12}$. Estimate the order of the method.

b. A quadrature method gives an error of 10^{-5} when $h = 10^{-2}$ and 10^{-13} when $h = 10^{-4}$. Estimate the order of the method.

c. A differential equation solver gives an answer $u(1) = 0.88888$ when $h = 0.1$, and $u(1) = 0.80666$ when $h = 0.01$, and $u(1) = 0.80600$ when $h = 0.001$. Estimate the order of the method.