Math 4329, Test II

Name _____

1. a. A table of values for $f(x)$ is:

x	$f(x)$
10	0.1000
11	0.0000
12	0.0000
13	0.0000

    Use cubic interpolation to estimate $f(10.5)$.

    b. If it is known that $|f^{iv}(x)| < 0.05$ for all x, obtain a reasonable bound on the error in your estimate of $f(10.5)$.

2. Use Taylor series expansions to determine the error in the approximation $u''(x) \approx \frac{u(x+2h)-2u(x+h)+u(x)}{h^2}$

3.   a. Find A, B which make the approximation

$\int_0^h f(x)dx \approx Ahf(0) + Bhf(\frac{2}{3}h)$

as high order as possible.

   b. What is the order of the global error, for this A,B?

4. (Note: you must do by hand and show your work.) Find the inverse of

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix},$$

5. A quintic spline interpolant is a function which is a polynomial of degree five or less in each interval $(x_{i-1}, x_i), i = 1, ..., N$ and passes through the points $(x_i, y_i), i = 0, ..., N$ and is continuous and has continuous first, second, third and fourth derivatives.

a. How many unknown coefficients need to be determined? (Hint: There are N intervals and the quintic has how many coefficients in each?)

b. How many interpolation conditions are there? (Hint: There are two interpolation conditions for each interval.)

c. How many continuity conditions are there? (Hint: s(x) is automatically continuous because of the interpolation conditions, so we only need to require that s', s'', s''', s^{iv} be continuous at each *interior* point—how many interior points are there?)

d. If you add the number of interpolation conditions (part b) and continuity conditions (part c), does this equal the number of unknowns (part a)? If not, what needs to be done to make the quintic spline interpolation problem have a unique solution?