Math 4329, Test II

Name ____________________________

1. If $P_2(x)$ is the second degree polynomial that interpolates to $f(x) = \frac{6}{1+x}$ at $x = 0, 0.1, 0.2$, find a reasonable bound on the error at $x = 0.15$.

2. Find A, B, C such that the approximation $u'(t) \approx \frac{Au(t)+Bu(t-h)+Cu(t-2h)}{h}$ is as high order as possible.

3. Find A, r which make the approximation

   $\int_{-1}^{1} f(x)dx \approx Af(-r) + Af(r)$

   as high degree of precision as possible (thus as high order as possible).

4. True or False:

   a. The experimental order of convergence is $O(h^3)$ if a quadrature rule yields errors of 0.0032 when $h = 0.1$ and 0.0002 when $h = 0.05$.

   b. The Gauss-Seidel iterative method (for $Ax = b$) is generally slower than the Jacobi method.

   c. The Jacobi iterative method (for $Ax = b$) converges only if the matrix is diagonal-dominant.

   d. Roundoff error is much more serious, in general, for derivative approximations than for integral approximations.

   e. Gaussian elimination, when applied to a general N by N linear system, requires $O(N^3)$ arithmetic operations.

   f. If $s(x)$ is a cubic spline, then s, s', s'' and s''' must be continuous everywhere.

   g. If a quadrature method is exact for all polynomials of degree n, its global error is $O(h^n)$ for general smooth functions.

   h. If a matrix A has condition number 10, we expect to lose about 10 significant digits in solving $Ax = b$ with Gauss elimination and partial pivoting.

5. a. Write out the Jacobi iteration, for the system

$$\begin{bmatrix} 4 & 3 & -1 \\ 1 & 4 & -1 \\ -1 & 2 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ -8 \end{bmatrix}$$

   Will it converge? Explain.

   b. Write out the Gauss-Seidel iteration, for this system