Math 4329, Test I

Name _____

1. a. If $f(x) = x^4 - 2x^3 + x + 3$, find the Taylor polynomial $T_3(x)$ of degree 3 which matches f, f', f'' and f''' at $a = 1$.

   b. Use the Taylor remainder formula to get a reasonable bound (in terms of x) on the error $|f(x) - T_3(x)|$ at x.

2. Computer A stores floating point numbers in a 128-bit word, which includes 1 sign bit, 21 bits for the exponent, and 106 bits for the mantissa. Computer B stores floating point numbers in a 128-bit word, with 1 sign bit, 13 bits for the exponent, and 114 bits for the mantissa.

   a. Which computer can handle larger numbers? Approximately what is the overflow limit for this computer?

   b. Which computer has higher accuracy? Approximately how many significant **decimal** digits of accuracy does this computer have?

3. Compute the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $0.03, 0.002$ and 0.00001.

4. Will the iteration $x_{n+1} = 4x_n(1 - x_n)$ converge when x_0 is sufficiently close to the root $r = \frac{3}{4}$? (Justify your answer theoretically, without actually iterating the formula.) If it converges, what is the order?

5. Show how Newton's method could be used to find $b^{p/q}$, where p, q are integers, and $b > 0$, without doing anything other than add, subtract, multiply and divide.

6. For the secant method, $e_{n+1} \approx Me_ne_{n-1}$. If the order of the secant method is α (ie, $e_{n+1} \approx Ce_n^\alpha$), show that α must satisfy $\alpha = 1 + 1/\alpha$.