Math 4329, Test I

Name _____

1. a. If $f(x) = ln(1+sin(x))$, find the Taylor polynomial $T_2(x)$ of degree 2 which matches f, f' and f'' at $a = 0$.

    b. Use the Taylor remainder formula to get a reasonable bound on the error $|f(x) - T_2(x)|$ at $x = 0.1$.

2. A certain computer stores floating point numbers in a 96-bit word. If a floating point number is written in normalized binary form $(1.xxxxx...\,_2 * 2^e)$, it is stored using one sign bit (0 if the number is positive), then $e + 2047$ is stored in binary in the next 12 bits, and then the mantissa xxxxx... is stored in the final 83 bits. Show exactly how the number -17.125 would be stored on this computer. Also, approximately how many **decimal** digits of accuracy does this machine have?

3. The polynomial $x^2 - x - 1$ has two roots, $r_1 = -0.618, r_2 = 1.618$. We can write $x^2 - x - 1 = 0$ in the form $x^2 = x + 1$, and then $x = 1 + \frac{1}{x}$ and try the iteration $x_{n+1} = 1 + \frac{1}{x_n}$.

Will this converge, for x_0 near $r_1 = -0.618$? Justify your answer **theoretically**. Will it converge near $r_2 = 1.618$?

4. Estimate the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $3 * 10^{-3}, 2 * 10^{-4}$ and $5 * 10^{-8}$.

5.   a. $r = \sqrt{a}$ is a root of $f(x) = x^2 - a = 0$. Write Newton's iteration for finding this root.

b. Given that, for Newton's method:

$$x_{n+1} - r = \frac{f''(c_n)}{2f'(x_n)}(x_n - r)^2$$

where c_n is between x_n and the root r, show that the iteration in 5a will converge provided $x_0 > \sqrt{a}/3$. (Hint: just show that x_1 will be closer to the root than x_0.)