Math 3323, Test II

Name _____

1. Do the given restrictions on vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

   in R^3 define subsets which are subspaces of R^3?

   a. $x_1 = x_2 x_3$ answer: no

   b. $x_1 - x_2 + x_3 = 2$ answer: no

   c. $x_1 + x_2 + x_3 = 0$ or $x_1 - x_2 + x_3 = 0$ answer: no

   d. $x_1 + x_2 + x_3 = 0$ and $x_1 - x_2 + x_3 = 0$ answer: yes

2. Identify the null space and range of the following matrices. If you can't identify the subspaces exactly, tell what you know about their dimensions:

   a. any nxn nonsingular matrix. answer: null space $= 0$, range $= R^n$

   b. The nxm zero matrix. answer: null space $= R^m$, range $= 0$

   c. any nxn singular matrix. answer: null space dim > 0, dim range $< n$

3. Which of the following subsets are subspaces? For those that are sub-
   spaces, give the dimension.

   a. The row space of the matrix

   $$A = \begin{bmatrix} 1 & 0 & 2 & 8 & -8 \\ 4 & 0 & 8 & 32 & -1 \\ 3 & 0 & 6 & 24 & -4 \end{bmatrix}$$

   answer: yes, dimension 2

   b. The set of solutions to $Ax = 0$ where A is the matrix in part a.
   answer: yes, dimension 3

   c. The column space of the matrix in part a.
   answer: yes, dimension 2

   d. The range of the matrix in part a.
   answer: yes, dimension 2

   e. The line in R^3, $x = t + 1, y = 2t, z = -t$.
   answer: no

   f. The set of vectors in R^5 such that $x_1 + x_2 + x_3 + x_4 + x_5 = 0$.
   answer: yes, dimension 4

4. For the matrix A of problem 3a:

   a. Find a basis for the subspace the row space of A.

   answer: (1,0,2,8,-8),(0,0,0,0,1)

b. Find a basis for the null space of A.

answer: (0,1,0,0,0),(-2,0,1,0,0),(-8,0,0,1,0)

5. If
$$u = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, w = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

a. Find $u \times v$.
answer: $(-6, 0, 3)$

b. Find the volume of a parallelopiped with edges u, v, w.
answer: 15

c. Find the equation of the plane through (1,2,3) parallel to the vectors u and v.
answer: $-2x + z = 1$

d. Find the equations of a line through $(1, 2, 3)$ perpendicular to the plane in part c.
answer: $x = 1 - 6t, y = 2, z = 3 + 3t$