Math 3323, Test II

Name _____

1. Do the given restrictions on vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

   in R^3 define subsets which are subspaces of R^3?

   a. $x_1 = 2x_2 x_3$ no

   b. $x_1 = x_2 + x_3$ or $x_1 = -x_2 + x_3$ no

   c. $x_1 = x_2 + x_3$ and $x_1 = -x_2 + x_3$ yes

2. Identify the null space and range of:

   a. Any nxn nonsingular matrix.
      answer: range $= R^n$, null space $= \theta$

   b. The nxn zero matrix.
      answer: range $= \theta$, null space $= R^n$

3. Which of the following subsets are subspaces? For those that are sub-spaces, give the dimension.

   a. The row space of the matrix

   $$A = \begin{bmatrix} 4 & 0 & 12 & 8 & -8 \\ 6 & 0 & 2 & 12 & -12 \\ 2 & 0 & 2 & 4 & -4 \end{bmatrix}$$

      yes, dim=2

   b. The set of solutions to $Ax = 0$ where A is the matrix in part a.
      yes, dim=3

c. The column space of the matrix in part a.   yes, dim=2

d. The line in R^3, $x = t, y = 2t, z = -t$.   yes, dim=1

e. The plane in R^3, $x + y + z = 1$.   no

4. Find a basis for the subspace spanned by the four vectors:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

answer: $[(1, 2, 1), (0, 1, -2)]$ is one answer

5. Find a basis for the null space of

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 0 \\ 2 & 1 & 5 & 4 & 0 \\ 1 & 2 & 4 & -1 & 0 \end{bmatrix},$$

answer: $[(-2, -1, 1, 0, 0), (-3, 2, 0, 1, 0), (0, 0, 0, 0, 1)]$ is one answer

6. Write the equations for:

   a. The line through the points (1,-1,2) and (3,3,3).

      answer: $x = 1 + 2t, y = -1 + 4t, z = 2 + t$

   b. The plane through (1,-1,2), perpendicular to this line.

      answer: $2(x - 1) + 4(y + 1) + (z - 2) = 0$

7. Find the area of the triangle with vertices $(1, 2, 2), (3, 3, 4), (5, 3, 3)$.

   answer: $\sqrt{41}/2$