Math 3323, Test I

Name ______________________________

1. If

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 1 \\ 3 & 6 & 9 & 0 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

find the general solution of $Ax = b$, and write in vector form.

answer: $(x_1, x_2, x_3, x_4) = (1, 0, 0, 0) + x_2(-2, 1, 0, 0) + x_3(-3, 0, 1, 0)$.

2. Solve problem 1 again, with the same A, but now $b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

answer: no solution

3. Suppose a linear system $Ax = b$ is solved, where A is an m by n matrix which has r nonzero rows after reduction to row echelon form. For each case described below, write "no", "unique" and/or "many" to indicate that no solution, a unique solution, or many solutions are possible. (More than one of these options may be possible).

   a. m=3, n=7, r=2 and b=0. (answer: many)
   b. m=7, n=3, r=3. (answer: no, unique)
   c. m=7, n=3, r=3 and b=0. (answer: unique)
   d. m=n, and the columns of A are independent. (answer: unique)
   e. m=n, b=0, and the columns of A are dependent. (answer: many)

4. Given that $1^2 + 2^2 + 3^2 + ... + n^2 = an + bn^2 + cn^3$, write a system of three equations in 3 unknowns to find a, b, c. (Don't solve the system.)

answer: $Ax = f$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}, x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, f = \begin{bmatrix} 1 \\ 5 \\ 14 \end{bmatrix}$$

5. Find the inverse of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 3 & 4 & 1 \end{bmatrix},$$

answer:

$$A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -4 & 5 & 1 \end{bmatrix},$$

6. Suppose v_1, v_2, v_3 are nonzero vectors such that $v_1^T v_2 = v_1^T v_3 = v_2^T v_3 = 0$. Show that the set of vectors v_1, v_2, v_3 is linearly independent.

answer: assume $av_1 + bv_2 + cv_3 = 0$, then $(av_1 + bv_2 + cv_3)^T(av_1 + bv_2 + cv_3) = a^2\|v_1\|^2 + b^2\|v_2\|^2 + c^2\|v_3\|^3 = 0$, and since the norms are all positive, this is only possible if $a = b = c = 0$.