

Math 2326, Test I

Name _____

For problems 1-3, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dt} = 3\sin(15t)(1 + y)$, with $y(0) = 0$

answer: $y(t) = e^{(1-\cos(15t))/5} - 1$

2. $\frac{dy}{dt} = \frac{t}{3y^2+2y-1}$, with $y(0) = 1$

answer: $y^3 + y^2 - y = \frac{1}{2}t^2 + 1$

3. $\frac{dy}{dx} + 2y = 6x^2$.

answer: $y = Ce^{-2x} + 3x^2 - 3x + \frac{3}{2}$

4. For the differential equation $y' = e^{3xy}$, $y(1) = 2$, take one step using Euler's method with $h = 0.001$, to approximate $y(1.001)$.

answer: $y(1.001) \approx 2.403$

5. Find all equilibrium points of $\frac{dy}{dt} = \cos(\pi y) - 1$ and classify each as a source, sink or node. Then tell what happens to $y(t)$ as $t \rightarrow \infty$, if $y(0) = -1$.

answer: $y =$ all even integers, all are nodes. $y \rightarrow -2$ as $t \rightarrow \infty$.

6. A 50 gallon tank is full to the brim with pure water, and 7 gallons/minute of a brine solution with 0.2 kg/gallon salt flows into it. Since the tank is full, 7 gallons/minute of well-mixed solution flows onto the ground. Find a differential equation, with initial condition, for $S(t)$, the number of kilograms of salt in the tank at time t . What happens to $S(t)$ as $t \rightarrow \infty$?

answer: $S'(t) = 1.4 - 0.14 S$, $S(0) = 0$. $S(\infty) = 10$