Math 2313, Test II

Name _____

1. If $f(x,y,z) = ln(x^2y^3z^4)$,

   a. Find the gradient of f at $(2,3,4)$.
   answer: $(1,1,1)$

   b. Find the derivative of f at $(2,3,4)$ in the direction of the vector $< 0,1,1 >$.
   answer: $\frac{2}{\sqrt{2}}$

   c. In what direction is the directional derivative smallest (most negative), at the point $(2,3,4)$?
   answer: $(-1,-1,-1)$

   d. Find the equation of the tangent plane to the surface $f(x,y,z) = ln(27648)$ at $(2,3,4)$.
   answer: $x + y + z = 9$

2. If $w(x,y) = x^4 + x^2y^2 + e^{xy}$, find:

   a. $\frac{\partial w}{\partial y} =$
   answer: $2x^2y + xe^{xy}$

   b. $w_{xx} + w_{yy} =$
   answer: $14x^2 + 2y^2 + (x^2 + y^2)e^{xy}$

3. If the temperature is $T(x,y) = x^2y^3 + ln(xy)$, what is the rate of change of temperature in a car at $(1,1)$, if the velocity of the car is $(\frac{dx}{dt}, \frac{dy}{dt}) = (-2,3)$?
answer: 6

4. Find the point on the surface $z = \sqrt{1-2x-4y}$ which is closest to the point $(-3,-5,0)$.

answer: $(-2,-3,\sqrt{17})$

5. If $(U_x, U_y, U_z) = (3,5,-1)$ at the point $(-1,0,0)$, which has spherical coordinates $\rho = 1, \phi = \frac{\pi}{2}, \theta = \pi$, find U_θ at this point. For spherical coordinates,

$$\begin{aligned} x &= \rho\ sin(\phi)cos(\theta) \\ y &= \rho\ sin(\phi)sin(\theta) \\ z &= \rho\ cos(\phi) \end{aligned}$$

answer: -5