Supplemental Exercises

<u>Feb. 7, 2019</u>

Solve the following problems using suitable random variables. Feel free to use the information on the handout about discrete and continuous probability distributions.

1. Two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability 1- p. The winner is the first team to win four games.

(a) What are the possible number of games played?

(b) What is the probability that one team wins by sweeping?

(c) What is the probability that team A wins in exactly 5 games?

(d) What is the probability that team B wins in no more than 6 games?

(e) What is the probability that 7 games are played?

2. I am new to basketball and so I am practicing to shoot the ball through the basket. Suppose the individual throws are independent of one another and probability of success at each throw is 0.2.

(a) What is that probability for me to fail on the first 3 throws?

(b) Given that I have failed the first 3 throws, what is the probability that I will fail the next 3 throws?

(c) On average, how many times would I fail on average before a success occurs?

3. A toll bridge charges $1.00 for passenger cars and $2.50 for other vehicles. Suppose that during daytime hours, 60% of all vehicles are passenger cars. If 25 vehicles cross the bridge during a particular daytime period, what is the resulting expected toll revenue?

4. The article “Methodology for Probabilistic Life Prediction of Multiple Anomaly Materials” (*AIAA J.* 2006: 787-793) proposes a Poisson distribution for the number of material anomalies occurring in a particular region of an aircraft gas-turbine

disk with parameter $\lambda = 4$. What is the probability that the number of anomalies exceeds its mean value by more than one standard deviation?

5. It has been suggested that the depth (cm) of the bioturbation layer in sediment in a certain region follow a uniform distribution on the interval (8, 20).

(a) What is the cdf of depth?

(b) What is the mean and variance of depth?

(c) What is the probability that the observed depth is within 2 standard deviations of the mean value?

6. A system consists of five identical components connected in series. As soon as one component fails, the entire system will fail. Suppose each component has a lifetime (in hours) that is exponentially distributed with $\lambda = 0.01$ and the components fail independently of one another. Is it likely that this system can be functional for at least 40 hours?

7. Suppose the time (in minutes) spent by a randomly selected student who uses a terminal connected to a local time-sharing computer facility has a gamma distribution with mean 20 and variance 80. What is the probability that a student spends between 20 and 40 minutes using the terminal?

8. A machine that produces ball bearings has initially been set so that the true average diameter of the bearings it produces is 0.500 in. A bearing is acceptable if its diameter is within 0.004 in of this target value. Suppose, however, that the setting has changed during the course of production, so that the bearings have normally distributed diameters with mean value 0.498 in. and standard deviation 0.002 in. What percentage of the bearings produced will not be acceptable?