Name: ___________________________

# STAT 3320 Class Exercise – August 29, 2019

Complete the following problems and turn in your work (please show all steps).

1. $6! = ?$

2. Expand the following:
   (a) $(1 - e^x)^2$
   (b) $(a + 1)^n$

3. Solve for $x$ in the following equations:
   (a) $x^2 - 3x + 2 = 0$
   (b) $3x^2 + 5x + 1 = 0$

4. Sum the following infinite series:

$$\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^n$$

5. $\frac{d}{dx}\left(x^2 - x + \sqrt{x} + 9\right) = \ ?$

6. Calculate the following integrals:
   (a) $\int_{-1}^{1} 3x^2 dx$
   (b) $\int_{0}^{\infty} xe^{x^2}\, dx$
   (c) $\int_{0}^{\infty} xe^{-x} dx$

7. (a) Sketch the graph of the function

$$f(x) = \begin{cases} 1 + x & -1 < x \leq 0 \\ 1 - x & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (b) $\int_{-\infty}^{\infty} f(x)dx = ?$

   (c) $\int_{-\infty}^{t} f(x)dx = ?$

Name: _________________________

# STAT 3320 Class Exercise – September 5, 2019

Complete the following problems and turn in your work (please show all steps).

1. Consider the random experiment of flipping a coin four times and record the sequence of outcomes as a 4-lettered string of H's (heads) and T's (tails). What is the sample space?

2. Consider the random experiment of flipping a coin four times and record the number of heads observed. What is the sample space?

3. If the coin used in Questions 1 and 2 is fair, what is the probability that exactly 2 heads are observed?

Name: ________________________

# STAT 3320 Class Exercise – September 10, 2019

Complete the following problems and turn in your work (please show all steps).

Only 1 in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that when an individual actually has the disease, a positive result will occur 99% of the time, whereas an individual without the disease will show a positive test result on 2% of the time.

(a) Intuitively, would you say that the diagnostic test is reasonably accurate?

(b) If a random adult is selected, what is the probability that this individual will have a positive test result? [Hint: Let the sample space $S$ be all the adults in the population. Partition $S$ into two events: $D$ = the event that the selected adult has the disease, and the complementary event $D'$. Let $B$ = the event that the selected adult has a positive test result. Apply the rule of total probability.]

Name: ________________________

# STAT 3320 Class Exercise – September 12, 2019

Complete the following problems and turn in your work (please show all steps).

The sample space $S$ contains four equally likely outcomes {a, b, c, d}. Let $A_1$, $A_2$, and $A_3$ be the events {a, d}, {b, d}, and {c, d} respectively.

(a) Show that the events $A_1, A_2, A_3$ are pairwise independent.

(b) Show that the events $A_1, A_2, A_3$ are NOT mutually independent.

(c) Does pairwise independence imply mutual independence? Does mutual independence imply pairwise independence?

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# STAT 3320 Class Exercise – September 17, 2019

Complete the following problems and turn in your work (please show all steps).

Consider the random experiment of rolling a fair die two times and record the number of dots shown on the uppermost face in each roll.

(a) What is the sample space?

(b) Let $A$ = the event that the first roll is 2, $B$ = the event that the second roll is 5. Are $A$ and B mutually independent? Are they mutually exclusive?

(c) Let $C$ = the event that the sum of the two rolls is 2, $D$ = the event that the sum of the two rolls is 5. Are $C$ and $D$ mutually independent? Are they mutually exclusive?

(d) Is it possible for two mutually independent events be also mutually exclusive?

Name: _________________________

# STAT 3320 Class Exercise – September 19, 2019

Complete the following problems and turn in your work (please show all steps).

Perform the random experiment of flipping a fair coin four times and record the sequence of heads (H) and tails (T) observed.

(a) Write down your outcome of the experiment.

(b) Let $X$ be the random variable representing the number of heads in the outcome. What is the value of $X$ for your outcome in part (a).

(c) Let $Y$ be the random variable representing the number of heads minus the number of tails in the outcome. What is the value of $Y$ for your outcome in part (a)?

(d) Let $Z$ be the random variable representing the reciprocal of the number of tails in the outcome. What is the value of $Z$ for your outcome in part (a)?

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# STAT 3320 Class Exercise – October 3, 2019

Complete the following problems and turn in your work (please show all steps).

Consider a binomial random variable $X$ with $n = 1$, and $p = 0.6$. [Note: any binomial random variable with $n = 1$ is called a Bernoulli random variable.]

(a) Show the probability distribution of $X$ in the form of a table, a probability histogram, and a probability mass function (pmf).

(b) Sketch the graph of its cumulative distribution function (CDF).

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# STAT 3320 Class Exercise – October 8, 2019

Complete the following problems and turn in your work (please show all steps).

Let $X \sim \text{Binom}(n = 3, p = 0.8)$

(a) Fill out the first three columns of the following table with the possible values of $X$, the corresponding probabilities $p(x)$, and the products $xp(x)$.

| $x$ | $p(x)$ | $xp(x)$ | $x^2p(x)$ | $(x - \mu)^2\, p(x)$ |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| sum | | | | |

(b) Obtain the mean $E(X)$. Let's call this $\mu$. Also write down the value of $\mu^2$ (i.e., $E(X)^2$).

(c) Fill out the fourth column of the table in (a) and find $E(X^2)$.
[Note that $E(X^2) \neq \mu^2$, which is larger?]

(d) Fill out the last column of the table in (a) and find the variance $V(X)$.

(e) Do you notice any relationship among $V(X)$, $E(X)$, and $E(X^2)$?

Name: ___________________________

# STAT 3320 Class Exercise – October 15, 2019

Complete the following problems and turn in your work (please show all steps).

(a) Sketch the graph of the function

$$f(x) = \begin{cases} 1 + x & -1 < x \leq 0 \\ 1 - x & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) Show that $f(x)$ is a legitimate probability density function (pdf). [Hint: You need to verify that $f(x)$ is always nonnegative, and that $\int_{-\infty}^{\infty} f(x)dx = 1$]

(c) Let $X$ be a random variable with $f(x)$ as it pdf. Find the mean and variance of $X$.

Name: ________________________

# STAT 3320 Class Exercise – October 22, 2019

Complete the following problems and turn in your work (please show all steps).

This exercise is about the standard normal distribution. Use the standard normal distribution table help you answer the questions below.

(a) Find the following percentiles for the standard normal ($Z$) distribution.

5th percentile =____________

10th percentile =____________

25th percentile =____________

50th percentile =____________

75th percentile =____________

90th percentile =____________

95th percentile =____________

In general, for $0 < p < 1$, what relationship do you see between the $(100p)$th and $100(1-p)$th percentiles?

(b) Write down the following critical values $z_\alpha$:

$z_{0.01}$ =___________

$z_{0.05}$ =___________

$z_{0.1}$ =___________

In general, for $0 < \alpha < 1$, what is the relationship between $z_\alpha$ and the $100(1- \alpha)$th percentile?

Name: _________________________

# STAT 3320 Class Exercise – October 29, 2019

Complete the following problems and turn in your work (please show all steps).

1. P.168 #45. A machine that produces ball bearings has initially been set so that the true average diameter of the bearings it produces is 0.5 inch. A bearing is acceptable if its diameters within 0.004 inch of this target value. Suppose, however, that the setting has changed during the course of production, so that the bearings have normally distributed diameters with mean value 0.499 inch and standard deviation 0.002 in. What percentage of the bearings produced will not be acceptable?

Name: ________________________

2. A certain market has both an express checkout line and a superexpress checkout line. Let $X$ denote the number of customers in line at the express checkout at a particular time of day, and let $Y$ denote the number of customers in line at the superexpress checkout at the same time. Suppose the joint pmf of $X$ and $Y$ is as given in the following table.

| | | $y$ | | | |
|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 |
| | 0 | 0.08 | 0.07 | 0.04 | 0.00 |
| | 1 | 0.06 | 0.15 | 0.05 | 0.04 |
| $x$ | 2 | 0.05 | 0.04 | 0.10 | 0.06 |
| | 3 | 0.00 | 0.03 | 0.04 | 0.07 |
| | 4 | 0.00 | 0.01 | 0.05 | 0.06 |

(a) What is the probability that the numbers of customers in the two lines are identical?

(b) What is the probability that the total number of customers in the two lines is exactly four? At least four?

(c) Calculate $E(XY)$.

(d) Write down the marginal distributions of $X$ and $Y$ at the appropriate margins of the joint pmf table.

(e) Find $E(X)$ and $E(Y)$.

(f) Find Cov $(X, Y)$. [Hint: Use the covariance formula.]