Name: Key

STAT 3320 Class Exercise – August 29, 2019

Complete the following problems and turn in your work (please show all steps).

1. $6! = ?$ $\quad 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

2. Expand the following:
   (a) $(1 - e^x)^2 = 1 - 2e^x + e^{2x}$
   (b) $(a+1)^n = a^n + \binom{n}{1}a^{n-1} + \binom{n}{2}a^{n-2} + \cdots + \binom{n}{n-1}a + 1$ (binomial expansion)

3. Solve for x in the following equations:
   (a) $x^2 - 3x + 2 = 0 \Leftrightarrow (x-2)(x-1) = 0 \Leftrightarrow x = 2$ or $x = 1$
   (b) $3x^2 + 5x + 1 = 0$ Quadratic formula: solutions for $ax^2+bx+c=0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
   $x = \frac{-5 \pm \sqrt{13}}{6}.$

4. Sum the following infinite series:

Sum of geometric series: $\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + \cdots = \frac{1}{1-r}$ provided that $-1 < r < 1$

$$\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^n = \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n$$
$$= \left(\frac{1}{5}\right)\left(\frac{4}{5}\right) \cdot \frac{1}{1-\left(\frac{4}{5}\right)} \quad \left[\text{Take } r = \left(\frac{4}{5}\right)\right]$$
$$= 4/5$$

5. $\frac{d}{dx}(x^2 - x + \sqrt{x} + 9) = ?$
   $2x - 1 + \frac{1}{2\sqrt{x}}$

6. Calculate the following integrals:
   (a) $\int_{-1}^{1} 3x^2 dx = [x^3]_{-1}^{1} = 1 - (-1) = 2$
   Let $u = x^2$ (b) $\int_0^{\infty} xe^{x^2} dx = \int_0^{\infty} \frac{1}{2} e^u du = \infty$
   Int. by parts $u = x$, $v = e^{-x}$ (c) $\int_0^{\infty} xe^{-x} dx = -\int_0^{\infty} x de^{-x} = -\left[xe^{-x}\Big|_0^{\infty} - \int_0^{\infty} e^{-x} dx\right] = -[0 - 1] = 1.$

7. (a) Sketch the graph of the function

$$f(x) = \begin{cases} 1 + x & -1 < x \le 0 \\ 1 - x & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (b) $\int_{-\infty}^{\infty} f(x) dx = ?$

   Come back to this later.

   (c) $\int_{-\infty}^{t} f(x) dx = ?$

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STAT 3320 Class Exercise – September 5, 2019

Complete the following problems and turn in your work (please show all steps).

1. Consider the random experiment of flipping a coin four times and record the sequence of outcomes as a 4-lettered string of H's (heads) and T's (tails). What is the sample space?

$\mathcal{S} = \{$HHHH, HHHT, HHTH, HHTT,
HTHH, HTHT, HTTH, HTTT,
THHH, THHT, THTH, THTT,
TTHH, TTHT, TTTH, TTTT$\}$

2. Consider the random experiment of flipping a coin four times and record the number of heads observed. What is the sample space?

$\mathcal{S} = \{0, 1, 2, 3, 4\}$

3. If the coin used in Questions 1 and 2 is fair, what is the probability that exactly 2 heads are observed?

Let A = event that exactly two heads are observed in the sample space in part (a).

A = {HHTT, HTHT, HTTH, THHT, THTH, TTHH}

Since it is a fair coin, each outcome in the sample space in part (a) occurs equally likely.

$\therefore$ By the rule of counting, $P(A) = \frac{n(A)}{n(\mathcal{S})} = \frac{6}{16} = \frac{3}{8}$.

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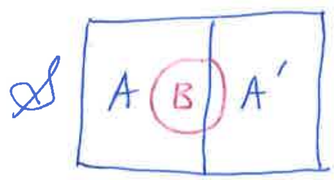
STAT 3320 Class Exercise – September 10, 2019

Complete the following problems and turn in your work (please show all steps).

Only 1 in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that when an individual actually has the disease, a positive result will occur 99% of the time, whereas an individual without the disease will show a positive test result on 2% of the time.

(a) Intuitively, would you say that the diagnostic test is reasonably accurate? Yes

(b) If a random adult is selected, what is the probability that this individual will have a positive test result? [Hint: Let the sample space S be all the adults in the population. Partition S into two events: A = the event that the selected adult has the disease, and the complementary event A'. Let B = the event that the selected adult has a positive test result. Apply the rule of total probability.]



$$\begin{aligned} P(B) &= P(B \cap A) + P(B \cap A') \\ &= P(B|A)P(A) + P(B|A')P(A') \\ &= (0.99)(0.001) + (0.02)(0.999) \\ &= 0.02097 \end{aligned}$$

(c) If a random adult is tested positive, what is the probability that the individual has the disease.

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} = \frac{(0.99)(0.001)}{0.02097} \quad \text{(Baye's Rule).}$$

$$= 0.4721$$

So there is only <5% chance that this individual has the disease

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STAT 3320 Class Exercise – September 12, 2019

Complete the following problems and turn in your work (please show all steps).

The sample space S contains four equally likely outcomes {a, b, c, d}. Let $A_1, A_2,$ and A_3 be the events {a, d}, {b, d}, and {c, d} respectively.

(a) Show that the events A_1, A_2, A_3 are pairwise independent.

$P(A_1) = P(A_2) = P(A_3) = \frac{1}{2}$

$P(A_1 \cap A_2) = P(\{d\}) = \frac{1}{4}$

$P(A_1 \cap A_3) = P(\{d\}) = \frac{1}{4}$

$P(A_2 \cap A_3) = P(\{d\}) = 1/4.$

So $P(A_1 \cap A_2) = P(A_1)P(A_2)$

$P(A_1 \cap A_3) = P(A_1)P(A_3)$

$P(A_2 \cap A_3) = P(A_2)P(A_3)$

$\therefore$ $A_1, A_2,$ & A_3 are pairwise independent.

(b) Show that the events A_1, A_2, A_3 are NOT mutually independent.

$P(A_1 \cap A_2 \cap A_3) = P(\{d\}) = 1/4.$

However $P(A_1)P(A_2)P(A_3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 1/8$

$P(A_1 \cap A_2 \cap A_3) \neq P(A_1)P(A_2)P(A_3)$

$\therefore$ A_1, A_2, A_3 are NOT mutually indep.

(c) Does pairwise independence imply mutual independence? Does mutual independence imply pairwise independence?

From above, pairwise independence $\not\Rightarrow$ mutual independence

but mutual independence $\Rightarrow$ pairwise independence by definition

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STAT 3320 Class Exercise – September 17, 2019

Complete the following problems and turn in your work (please show all steps).

Consider the random experiment of rolling a fair die two times and record the number of dots shown on the uppermost face in each roll.

(a) What is the sample space? $\mathcal{S} = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2) - - - - - (2,6),$
$\vdots \quad \vdots$
$(6,1), - - - - (6,6)\}$.

(b) Let A = the event that the first roll is 2, B = the event that the second roll is 5. Are A and B mutually independent? Are they mutually exclusive?

$P(A) = 1/6$, $P(B) = 1/6$. since $A = \{(2,1) \ldots (2,6)\}$, $B = \{(1,5) \ldots (6,5)\}$

$A \cap B = \{(2,5)\}$ $P(A \cap B) = \frac{1}{36} = P(A)P(B)$

$\therefore$ A and B are independent.

They are not mutually exclusive because $A \cap B \neq \phi$

(c) Let C = the event that the sum of the two rolls is 2, D = the event that the sum of the two rolls is 5. Are C and D mutually independent? Are they mutually exclusive?

$C = \{(1,1)\}$, $P(C) = 1/36$

$D = \{(1,4), (2,3), (3,2), (4,1)\} = 1/9$

$P(C \cap D) = 0$ since $C \cap D = \phi$.

So $P(C \cap D) \neq P(C)P(D)$. $\therefore$ they are not independent.

They are mutually exclusive because $C \cap D = \phi$.

(d) Is it possible for two mutually independent events be also mutually exclusive?

Yes, but only when at least one of the events has 0 prob.

mutually exclusive $\Rightarrow P(A \cap B) = 0$.

independent $\Rightarrow P(A \cap B) = P(A)P(B)$

$\therefore$ $P(A)P(B) = 0$, so $P(A) = 0$ or $P(B) = 0$.

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STAT 3320 Class Exercise – September 19, 2019

Complete the following problems and turn in your work (please show all steps).

Perform the random experiment of flipping a fair coin four times and record the sequence of heads (H) and tails (T) observed. Answers differ. The following is just an example.

(a) Write down your outcome of the experiment.

HTHH

(b) Let X be the random variable representing the number of heads in the outcome. What is the value of X for your outcome in part (a).

For the particular outcome HTHH, $X = 3$

(c) Let Y be the random variable representing the number of heads minus the number of tails in the outcome. What is the value of Y for your outcome in part (a)?

$$Y = \underset{\#H's}{3} - \underset{\#T's}{1} = 2$$

(d) Let Z be the random variable representing the reciprocal of the number of tails in the outcome. What is the value of Z for your outcome in part (a)?

$$Z = \frac{1}{\#T's} = \frac{1}{1} = 1.$$