MATRIX ALGEBRAS WITH MULTIPLICATIVE DECOMPOSITION PROPERTY

PIOTR J. WOJCIECHOWSKI

Abstract. We will continue our studies of subalgebras of $M_n(\mathbb{R})$ that have the MD property. Recall that a directly entry-wise ordered algebra $\mathcal{A}$ of $M_n(\mathbb{R})$ satisfies the *Multiplicative Decomposition* property if for every $0 \leq A, B, C \in \mathcal{A}$ such that $C \leq AB$, there exist $0 \leq A', B' \in \mathcal{A}$ such that $C = A'B'$. Previously we proved that every such algebra embeds into some $\mathcal{A}_\sigma$, i.e. into an algebra of matrices with a given signature (for every i, the i^{th} row or the i^{th} column has at most one nonzero entry and it is at the i^{th} place).

We will show now that in the diagonal part of $\mathcal{A}$ there is a matrix D with the property that if $d_{ii} \neq 0$ and $d_{jj} \neq 0$ for some $i \neq j$, then for every matrix $A \in \mathcal{A}$, $a_{ij} = 0$.

We conjecture that a directly ordered subalgebra $\mathcal{A}$ of $M_n(\mathbb{R})$ has the MD property if and only if it is a subalgebra of some $\mathcal{A}_\sigma$ and it satisfies the above condition.

Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, TX 79968

E-mail address: piotrw@utep.edu