Hyperbolic realization of graphs and graph pairs

Luis Valdez-Sanchez

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Abstract: Let M be a compact, connected hyperbolic 3-manifold with a torus boundary component T_0. By Thurston's hyperbolic Dehn filling theorem, with finitely many exceptions, any Dehn filled manifold $\mathcal{M} = M \cup_{T_0} S^1 \times D^2$ is also a hyperbolic manifold. Determining how many exceptional cases there are for a particular 3-manifold is a crucial part of the classification problem of 3-manifolds in general, greatly advanced by the recent proof of Thurston's Geometrization Conjecture by G. Perelman.

One way $\mathcal{M} = M \cup_{T_0} S^1 \times D^2$ may not be hyperbolic is if it contains an incompressible closed torus $\widehat{T}$, ie if $\mathcal{M}$ is *toroidal*. If there is a different Dehn filling $\mathcal{M}' = M \cup_{T_0} (S^1 \times D^2)'$ of M which is also toroidal, with incompressible torus $\widehat{T}'$, it may be possible to obtain information about the homeomorphism type of M from the *graphs of intersection* in M between $T = \widehat{T} \cap M$ and $T' = \widehat{T}' \cap M$.

In this talk I will present conditions under which abstract graph pairs on punctured tori T, T' can be realized by embeddings $T, T' \subset (M, T_0)$ in hyperbolic 3-manifolds M. Such conditions seem to cover all known examples of graph pairs in the literature.