MATRIX ALGEBRAS WITH MULTIPLICATIVE DECOMPOSITION PROPERTY II

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Abstract. We say that a directly entry-wise ordered algebra $\mathcal{A}$ of $n \times n$ matrices satisfies the *Multiplicative Decomposition* property if for every $0 \leq A, B, C \in \mathcal{A}$ such that $C \leq AB$, there exist $0 \leq A', B' \in \mathcal{A}$ such that $C = A'B'$. Let a *signature* be an n-element sequence $\sigma = (s_i)$, where $s_i = R$ or C. We say that the matrix A has the signature σ if for $i = 1, \ldots, n$, $a_{ij} = 0$ for every $j \neq i$ provided that $s_i = R$, and $a_{ji} = 0$ for every $j \neq i$ provided that $s_i = C$. The collection of all matrices with a given signature forms an algebra with the MD property. We will call such an algebra a *signature* algebra and denote it by $\mathcal{A}_\sigma$. We prove the embedding theorem: *Every matrix algebra with the MD property embeds in some* $\mathcal{A}_\sigma$. We conjecture more, that every matrix algebra with the MD property is isomorphic to a direct sum of full specific subalgebras of $\mathcal{A}_\sigma$ and $\mathbb{R}^k$ for some k. We prove the conjecture in case of algebras of matrices having at most one non-diagonal entry.

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