MATRIX ALGEBRAS WITH MULTIPLICATIVE DECOMPOSITION PROPERTY

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Abstract. The Riesz Decomposition property is fundamental in the theory of lattice-ordered groups and related topics. In case of a ring, it applies to its underlying abelian group structure. Some authors however (eg. Beukers, Huijsmans, DePagter, Dai, DeMarr), have investigated the analogue of the decomposition within the multiplicative structure of a ring and considered the so-called *Multiplicative Decomposition* or *MD* property. We will recall some of the results for the ring $C(X)$ of real continuous functions and, more generally, for any Dedekind σ-complete partially-ordered algebra. Motivated by this line of research we proceed to a characterization of matrix algebras (i.e. subalgebras of $M_n(\mathbb{R})$) that have the MD property. We introduce an idea of an *RC-signature* of a matrix, and will prove that all matrices from a given algebra with the MD property have the same signature. We prove that within $M_2(\mathbb{R})$ a subalgebra has the MD property if and only if it is $U_2(\mathbb{R})$ or $L_2(\mathbb{R})$ or is diagonal. We conjecture the result generalizes to the $n \times n$ case, so that *a matrix algebra has the MD property if and only if it is an algebra of all matrices with a given RC-signature or is diagonal.*

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