Colloquium

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Friday, February 27, 2015 at 3pm in Bell Hall 143

Topological Boolean Algebras and Heyting Algebras

A topological algebra is an algebra, such a a group, or lattice, that is equipped with a topology that makes the algebraic operations continuous. There is a deep and extensive literature on topological algebras, and especially on topological groups. The famous Pontryagin duality and Peter-Weyl theorems are notable instances.

It is well known that each compact Hausdorff Boolean algebra is isomorphic and homeomorphic to the Boolean algebra of maps from a set X to the 2-element Boolean algebra with product topology. This can be established using Pontryagin duality by considering a Boolean algebra as a Boolean ring, but this proof gives little insight into why the result is true. Here we give a more direct proof that illuminates the combinatorial nature of this result. This is based on Dikranjan's treatment of the Peter-Weyl theorem.

We also show that a corresponding result holds for compact Hausdorff Heyting algebras. Here there is no recourse to an underlying group structure. Rather, the result for the Boolean case is bootstrapped up.