Math 5330 Final Exam

Name ____________________________

1. a. A bicycle manufacturer builds one-, three- and ten-speed models. The models need 20,30 and 40 units of steel, respectively, and 12,21 and 16 units of aluminum, and the company has available 91,800 units of steel and 42,000 units of aluminum. How many of each model should be made to maximize the profit, if the company makes \$8 per one-speed, \$12 per three-speed, and \$24 per ten-speed? What is the maximum possible profit?

b. Write the (symmetric) dual for the above primal problem. Would the dual be easier to solve than the primal using the simplex method? Explain.

c. The dual solution is y = (0.6,0.0). Explain the significance of the two components of y, for this application.

2. Two factories have 300 and 400 cars, three dealers need 100, 120 and 60 delivered to them. The cost C_{ij} to transport each car from factory i to dealer j is: $C_{11} = 100, C_{12} = 150, C_{13} = 180, C_{21} = 250, C_{22} = 240, C_{23} = 180$. Set up the initial simplex tableaux for this problem (but don't solve!), using slack and artificial variables as needed.

3. What is the order of work for each of the following? Assume all matrices are N by N and full unless otherwise stated, and assume advantage is taken of any special structure mentioned.

   a. One Jacobi iteration for finding the eigenvalues of a symmetric matrix A.

   b. The solution of $Ax = b$ using Gaussian elimination, if A is tridiagonal except A_{1N} and A_{N1} are also nonzero.

   c. One QR iteration, if A is upper Hessenberg.

   d. One QR iteration, if A is symmetric and tridiagonal.

   e. One power method iteration.

   f. Normal equations method, to solve $min\|Ax - b\|_2$. Assume A is $2N$ by N.

   g. A fast Fourier transform, that is, multiplication Ax, where $A_{j,k} = exp(i2\pi(j-1)(k-1)/N)$, and N is a power of 2.

   h. A slow Fourier transform, that is, multiplication Ax using the usual matrix multiplication formula.

   i. One simplex step, for solving max $c^T x$ with $Ax \leq b, x \geq 0$.

   j. Solution of $Ax = b$ using Gaussian elimination with partial pivoting, if A is banded, with bandwidth $\sqrt{N}$.

   k. Back substitution, if A is as in part (j).

   l. The orthogonal transformation of a full matrix to a similar upper Hessenberg matrix.

   m. Solution of $Ax = b$ if an LU decomposition of A is known and used.

   n. Solution of $min\|Ax - b\|_2$ if a QR decomposition of A is known and used. Assume A is $2N$ by N.

   o. One Gauss-Seidel iteration, for solving $Ax = b$.

4. True or False

   a. It is possible to reduce a general 5 by 5 symmetric matrix to diagonal form in a finite number of steps, using the Jacobi (eigenvalue) algorithm.

b. If λ is the largest eigenvalue of $(A - pI)^{-1}$, then $p - \frac{1}{\lambda}$ is the eigenvalue of A closest to p.

c. If A is symmetric and positive definite, its singular values are the same as its eigenvalues.

d. The simplex method is guaranteed to converge in at most $3N$ steps, where N=number of unknowns, assuming there is a solution.

e. If A is diagonal dominant, both the Jacobi and Gauss-Seidel iterations for solving $Ax = b$ are guaranteed to converge, for any starting guess.

f. The revised simplex method is generally faster (than the standard simplex method) when the constraint matrix is sparse and has many more equations than unknowns.

g. The fast Fourier transform is more efficient when N is a power of 2, than when N is prime.

h. If $x = A^T(AA^T)^{-1}b$, then x is the solution to min $\|Ax - b\|_2$, assuming the indicated inverse exists.

i. If $x = (A^TA)^{-1}A^Tb$, then x is the solution to $Ax = b$ of minimum 2-norm, assuming the indicated inverse exists.

j. If A has a condition number of 10, we should expect to lose about 10 significant decimal digits in solving $Ax = b$ with Gaussian elimination and partial pivoting.

k. The SOR method is guaranteed to converge if A is symmetric and positive definite, and $0 < \omega < 2$.

l. The generalized eigenvalue problem $Ax = \lambda Bx$ may have complex eigenvalues, even if A and B are symmetric.

m. If all eigenvalues have distinct absolute values, the QR iteration will converge to triangular form, even if you start from a full matrix.

n. The revised simplex method is more efficient than the usual simplex method, for transportation problems.

o. A fast Fourier tranform routine can be used to find an inverse Fourier transform, with little additional effort.