Math 5330, Test II

Name _____

1. a. Find a QR decomposition of

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & -5 \end{bmatrix}$$

   b. Use this QR decomposition to find $min\|Ax - b\|_2$, where $b = (1, 2, -1)$.

   c. Find $min\|Ax - b\|_2$ using the normal equations method.

1

2. (Answer either (a) OR (b), and (c) and (d)). If

$$A = \begin{bmatrix} 12 & 5 \\ 5 & 12 \end{bmatrix}$$

a. Do one QR iteration on A.

b. Do one LR iteration on A.

c. Use the power method to find the largest eigenvalue (in absolute value) of A, starting with $x_0 = (2, 1)$. (Hint: the eigenvalues are integers.)

d. Repeat (c) but start with $x_0 = (1, -1)$. Explain why the answer is not the same as in (c). What would you expect to happen if you started with $x_0 = (1, -1.001)$?

3. (Answer either (a) or (b)). If

$$A = \begin{bmatrix} 1 & -12 & 5 \\ -12 & 2 & 3 \\ 5 & 3 & 4 \end{bmatrix}$$

   a. Find an orthogonal matrix Q such that QAQ^{-1} is upper Hessenberg.

   b. Find an elementry matrix M such that MAM^{-1} is upper Hessenberg.

4. Find the eigenvalues of the quasitriangular matrix:

$$A = \begin{bmatrix} 2 & -12 & 5 & -7 \\ 0 & 12 & 5 & 3 \\ 0 & 5 & 12 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$