Math 5330, Test II

Name ____________________________

Work any 5 problems

1. Given that the QR decomposition of A is

$$Q = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{bmatrix}, R = \begin{bmatrix} 2 & 5 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$

use this to find x which minimizes $\|Ax - b\|_2$, where $b = (2, -3, 4)$.

2. Find the straight line $f(x) = mx + b$ which most nearly interpolates the points $(0, -1), (2, 2), (3, 3), (5, 4)$ in the least squares sense.

3. Prove the following:

   a. If $A^T Ax = A^T b$, then x minimizes $\|Ax - b\|_2$.

b. $I - \frac{2ww^T}{w^T w}$ is orthogonal, for any vector $w \neq 0$.

4. Find all eigenvalues of the pseudo-triangular matrix

$$\begin{bmatrix} -3 & 7 & 8 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & -4 & 2 & 7 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

5. If the Jacobi iteration $A_{n+1} = Q_n^T A_n Q_n$, where $A_1 = A$ converges to diagonal form in, say, 10 iterations, so that $A_{11} \approx D$, what are the eigenvalues of A, and what are the eigenvectors?

6. a. Find an orthogonal matrix Q such that QAQ^{-1} is upper Hessenberg, if

$$A = \begin{bmatrix} 2 & 4 & -3 \\ 4 & 1 & 7 \\ -3 & 7 & 1 \end{bmatrix}$$

b. Is QAQ^{-1} symmetric (note: you need not actually find A)?

7. a. Do one complete iteration of the LR method, starting with

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -3 & -2 & 1 \\ 0 & 4 & -8 \end{bmatrix}$$

b. Is the new matrix still tridiagonal?

c. If you had done a QR iteration instead of LR, would the new matrix still be tridiagonal?