Math 5330, Test I

Name _____

1. If

$$A = \begin{bmatrix} -4 & 2 & 1 \\ 0 & 3 & 1 \\ 8 & 2 & 3 \end{bmatrix}$$

find a lower triangular matrix L, and an upper triangular matrix U such that $A = LU$.

2. An N by N band matrix has $\sqrt{N}$ non-zero diagonals below the main diagonal and the same number above. If N is large, approximately how many multiplications are done:

   a. during the forward elimination, if no pivoting is done?

   b. during the forward elimination, if partial pivoting is done?

   c. during back substitution, if no pivoting is done?

   d. during back substitution, if partial pivoting is done?

3. A MATLAB program which solves a symmetric linear system, with no pivoting, using Gauss-Jordan, does most of its work in the loops:

```
      for I=1:N
%                    ELIMINATE ELEMENTS ABOVE DIAGONAL IN COLUMN I
         for J=1:I-1
            for K=I:N
               A(J,K) = A(J,K) - LJI*A(I,K);
            end
         end
%                    ELIMINATE ELEMENTS BELOW DIAGONAL IN COLUMN I.
%                    TAKE ADVANTAGE OF SYMMETRY HERE.
         for J=I+1:N
            for K=J:N
               A(J,K) = A(J,K) - LJI*A(I,K)
            end
         end
      end
```

   Approximately how many multiplications are done (show work)? How does this compare to Gaussian elimination for a nonsymmetric system?

4. Prove that $\frac{\|\Delta x\|}{\|x\|} \leq cond(A)\frac{\|\Delta b\|}{\|b\|}$ if $Ax = b$ and $A(x + \Delta x) = b + \Delta b$.

5. Would you expect the Jacobi iterative method to converge, when used to solve:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

What about the Gauss-Seidel method? Justify your answers theoretically, that is, without actually taking any iterations.

6. Define:

   a. orthogonal matrix

   b. lower Hessenberg matrix

   c. tridiagonal matrix

   d. positive definite matrix

   e. $\|x\|_1$, if x is a vector

   f. $\|A\|_2$, if A is a matrix