Tuesday, December 3, 2019 1:52 PM

$$2x_1^{(1)} + 5x_2^{(2)} = 2$$

$$2(-4/3) + 5x_2^{(2)} = 2$$

$$= \frac{1}{5}\left(2 + \frac{8}{3}\right)$$

$$\boxed{x_2^{(2)} = \frac{14}{15}}$$

$$N = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \qquad P = N - A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$$

$$N^{-1}P = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/3 \\ -2/5 & 0 \end{bmatrix}$$

$$\|N^{-1}P\| = 1 < 1$$

Gsiedel :

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$$ A is D.D $\Rightarrow$ Gauss Seidel will converge!

G.seidel Iterations :

$$\boxed{x_1^{(1)} = -4/3}$$

$$x_2^{(1)} = \frac{1}{5}\left(\cancel{2}\,1 - 2x_1^{(1)}\right)$$

$$\boxed{x_2^{(1)} = \frac{2}{5}\left(1 + 4/3\right) = \frac{14}{15}}$$

$$x_1^{(2)} = 1/3\left(-4 + x_2^{(1)}\right) = 1/3\left(-4 + \frac{14}{15}\right)$$

$$= 1/3\left(\frac{-46}{15}\right) = -46/45$$

Jn

$$2x_1^{(2)} + 5x_2^{(2)} = 2$$

$$x_2^{(2)} = \frac{1}{5}\left(2 - 2\left(\frac{-46}{45}\right)\right)$$

$$x_2 = \frac{1}{5}\left(- \quad \left(\quad \frac{}{45} \right) \right)$$

$$= \frac{2}{5}\left(\frac{45+46}{45}\right) = \frac{182}{225}$$

I 1.

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 6x_2 + 8x_3 &= 3 \\ 6x_1 + 8x_2 + 18x_3 &= 5 \end{aligned}$$

$$A \quad x \quad = b$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

2. Use LU factorization of A to solve $Ax = b$.

Get $A = \boxed{LU}$ and then solve

$$Ly = b \longrightarrow Ux = y$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix} \xrightarrow[R_3 \to R_3 - m_{32}R_2]{\substack{R_2 \to R_2 - m_{21}R_1 \\ R_3 \to R_3 - m_{31}R_1}} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 6 \\ 0 & 0 & 9 \end{bmatrix}}_{U}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 6 & 1/2 & 1 \end{bmatrix}$$

$$m_{21} = \frac{a_{21}}{a_{11}} = 2$$

$$m_{31} = a_{31}/a_{11} = 6$$

Solve $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ \end{bmatrix}$$

$m_{31} = a_{31}/a_{11} = 6$

$$\begin{bmatrix} 2 & 1 & 0 \\ 6 & 1/2 & 1 \end{bmatrix}\begin{bmatrix} y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$y_1 = 1$

$2y_1 + y_2 = 3 \to y_2 = 1$

$6y_1 + \tfrac{1}{2}y_2 + y_3 = 5$

$6 \cdot 5 + y_3 = 5 \Rightarrow y_3 = -3/2$

$Ly = b$

$$y = \begin{bmatrix} 1 \\ 1 \\ -3/2 \end{bmatrix} \longrightarrow \quad Ux = y$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 6 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -3/2 \end{bmatrix}$$

$x_1 + x_2 + x_3 = 1$

$4x_2 - 1 = 1 \Rightarrow \boxed{x_2 = 1/2} \leftarrow 4x_2 + 6x_3 = 1$

$\boxed{x_3 = -1/6} \leftarrow 9x_3 = -3/2$

$x_1 + \underbrace{1/2 - 1/6}_{\frac{6-2}{12} = 1/3} = 1 \Rightarrow x_1 = 1 - 1/3 = 2/3$

$x_1 = 2/3 \quad x_2 = 1/2, \quad x_3 = -1/6.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 6 & 8 & 3 \\ 6 & 8 & 18 & 5 \end{array}\right] \dashrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 4 & 6 & 1 \\ 0 & 0 & 9 & -3/2 \end{array}\right]$$

$x_1 + x_2 + x_3 = 1$

$4x_2 + 6x_3 = 1$

$9x_3 = -3/2$

exactly same as y

Final Review:

Chap1 → * function evaluation

$\sqrt{3}$ → Taylor polynomial $f(x) = \sqrt{x}$

↳ Taylor poly.

error analysis:

$$|f(x) - P_n(x)| \le \frac{|(x-a)^{n+1}|}{(n+1)!} f^{(n+1)}(c_x)$$

provide c_x that give max value.

* Floating Point Representatn

mathematically same but numerically different!

* use Taylor expansion

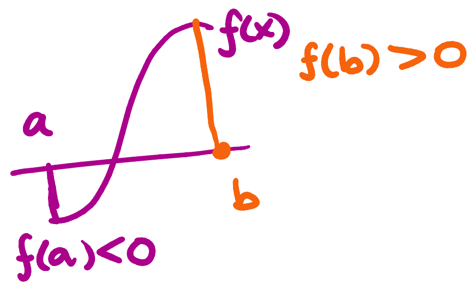
$\sqrt{x+1} - \sqrt{x}$

Root Finding

Bisection

$f(x) = 0$

$\boxed{e^x \cos x = 1}$



find the # of midpoints that need to be calculated to have $|\alpha - c_n| < 10^{-5}$?

Newton's method (remember formula) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

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secant method replaces $f'(x_n)$ with $\frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$

$f(x) = x^3 - 2$

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} = x_n - \frac{(x_n^3 - 2)(x_n - x_{n-1})}{x_n^3 - x_{n-1}^3}$$

$(x_n - x_{n-1})(x_n^2 + x_{n-1}^2 + x_n x_{n-1})$

$$x_{n+1} = x_n - \frac{(x_n^3 - 2)\cancel{(x_n - x_{n-1})}}{\cancel{(x_n - x_{n-1})}(x_n^2 + x_{n-1}^2 + x_{n-1}x_n)}$$

$$= \frac{x_n(x_n^2 + x_{n-1}^2 + x_{n-1}x_n) - (x_n^3 - 2)}{(x_n^2 + x_{n-1}^2 + x_{n-1}x_n)}$$

$\cancel{x_n^3} + x_n x_{n-1}^2 + x_{n-1}x_n^2 - \cancel{x_n^3} + 2$

$$x_{n+1} = \frac{x_n x_{n-1}^2 + x_{n-1}x_n^2 + 2}{x_n^2 + x_{n-1}^2 + x_{n-1}x_n}$$

- Newton's → quadratic
- secant → superlinear
- Bisection → linear

$$x^2 - 5 = 0$$

mult. by c

$$x + c(x^2 - 5) = 0 + x$$

c could be any real number

$$\boxed{x_{n+1} = \underbrace{x_n + c(x_n^2 - 5)}_{g(x_n)}}$$

$-1 < g'(\alpha) < 1$

$g(x) = x + c(x^2 - 5)$ $g'(x) = 1 + 2cx$

$-1 < 1 + 2c\alpha < 1$

$-1 < g'(\alpha) < 1$

$g(x) = x + \check{c}(x^2 - 5)$ $g'(x) = 1 + 2cx$

$-1 < g'(\alpha) = 1 + 2c\alpha < 1$