# 4329-II exam 03

Tuesday, December 3, 2019 1:28 PM

IV. Interpolation problem

III. For linear system $Ax=b$,

consider $x^{(k+1)} = b + M x^{(k)}$ $k=0,1,\ldots$

$$M = \begin{bmatrix} \alpha & -0.5 \\ 0 & 2\alpha \end{bmatrix},$$ $\alpha$ is unknown.

under what conditions on $\alpha$ will the iterative method $x^{(k+1)} = b + Mx^{(k)}$ converge?

From conv. analysis,

$$N x^{(k+1)} = b + P x^{(k)}$$

$\|N^{-1}P\| < 1 \Rightarrow$ formula converges.

$N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $P = \begin{bmatrix} \alpha & -0.5 \\ 0 & 2\alpha \end{bmatrix}$

$$\|N^{-1}P\| = \left\| \begin{bmatrix} \alpha & -0.5 \\ 0 & 2\alpha \end{bmatrix} \right\|$$

$$= \max\{ |\alpha| + 0.5,\ 2|\alpha| \} < 1$$

$|\alpha| + 0.5 < 1$ and $2|\alpha| < 1$

$\Downarrow$ $\Downarrow$

$|\alpha| < 1 - 0.5 = 0.5$ $|\alpha| < 1/2 = 0.5$

$|\alpha| < 1 - 0.5 = 0.5$ $\qquad |\alpha| < 1/2 = 0.5$

$$-0.5 < \alpha < 0.5$$

II. $\quad$ ① $3x_1 - x_2 = -4$

$\qquad 2x_1 + 5x_2 = 2$ ②

Apply 2 iterations of Jacobi & Gauss Seidel with $x^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Do we have convergence?

Jacobi: $x^{(1)} = \begin{bmatrix} -4/3 \\ 2/5 \end{bmatrix} \to x^{(2)} = \begin{bmatrix} -18/15 \\ \phantom{0} \end{bmatrix}$

$3x_1^{(2)} - 2/5 = -4 \Rightarrow x_1^{(2)} = \frac{1}{3}(-4 + 2/5)$

$= -18/15$

$2x_1^{(1)} + 5x_2^{(2)} = 2$

b