Math 1312

Tuesday, December 3, 2019 3:05 PM

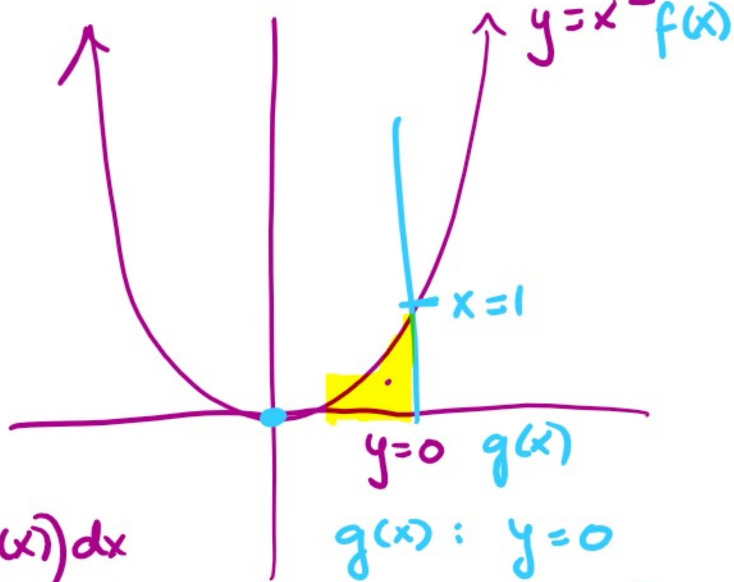
$M = \rho \int_0^1 x^2 - 0 \, dx = \rho/3$

$M_y = \rho \int_0^1 x(f(x) - 0) \, dx$

$= \rho \int_0^1 x^3 \, dx = \rho/4$

$M_x = \rho \int_0^1 \left(\frac{f(x)+g(x)}{2}\right)(f(x)-g(x)) \, dx$

$= \frac{\rho}{2} \int_0^1 x^4 \, dx = \rho/10 \rightarrow \left(\frac{\rho/4}{\rho/3}, \frac{\rho/10}{\rho/3}\right) = (3/4, 3/10)$



$g(x): y = 0$

$f(x): y = x^2$

v

#2

$-2, 3/4, 4/9, 5/16, \ldots \rightarrow \cos n\pi \left(\frac{n+1}{n^2}\right)$

$3, 5, 3, 5 \ldots \rightarrow 4 + (-1)^n$

$1, 1/2, 1/6, \ldots \rightarrow 1/n!$

#3 $\sum_{n=1}^{\infty} \frac{n+1}{n^4+2}$ $\quad \frac{a_n}{b_n} = \frac{n+1}{n^4+2} \times \frac{n^4}{n} \rightarrow \frac{1+1/n}{1+2/n^4}$

$\rightarrow 1$

$\sum b_n \rightarrow ?$

$\frac{1}{b_n} = \frac{n^4}{n} \rightarrow b_n = 1/n^3$ p-series with $p = 3 > 1$

$\frac{1}{b_n} = \frac{n^4}{n} \rightarrow b_n = 1/n^3$ p-series
p=3>1
converges.

3(b) $a_n = \frac{1}{3^n+1}$ $\sum_{n=1}^{\infty} 1/3^n$

$$3^n + 1 > 3^n$$

$$\frac{1}{3^n+1} < 1/3^n$$

$\sum 1/3^n$ is Geom. series with ratio $1/3 < 1$

$\Rightarrow \sum 1/3^n$ converges $\Rightarrow \sum 1/3^n+1$ also converges.

4)(a) $\sum_{n=2}^{\infty} \frac{2^n}{3^{n+1}}$ $\frac{2^2}{3^3} + \frac{2^3}{3^4} + \frac{2^4}{3^5} + \cdots$

$a + ar + ar^2 + \ldots$

$a = 4/27$ $r = \frac{2^3}{3^4} \div \frac{2^2}{3^3}$

$= \frac{2^3}{3^4} \times \frac{3^3}{2^2} = 2/3$

$a + ar + ar^2 + ar^3 + \ldots = a/1-r$ $|r|<1$

Sum $= (4/27) \Big/ 1-2/3 = \frac{4}{27} * 1/3 = \frac{4}{9}$

∞

(b) $\sum_{n=0}^{\infty} 1/5^n = 1 + 1/5 + 1/5^2 + 1/5^3 + \cdots$

$a \quad ar \quad ar^2 \quad ar^3 + \cdots$

$r = 1/5 < 1 \longrightarrow \frac{a}{1-r}$

$= \frac{1}{1-1/5} = 5/4$

#5 (a) $\sum_{n=1}^{\infty} \frac{(n+1)!}{2^n}$ $\quad a_n = \frac{(n+1)!}{2^n}$

Ratio Test $\quad \frac{a_{n+1}}{a_n} = \frac{((n+1)+1)!}{2^{n+1}} \div \frac{(n+1)!}{2^n}$

$\frac{a_{n+1}}{a_n} = \frac{(n+2)!}{2^{n+1}} * \frac{2^n}{(n+1)!} = \frac{(n+2)(n+1)!}{2\,(n+1)!} = \frac{n+2}{2}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty \Rightarrow \sum a_n$ diverges. (Ratio test)

(b) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ $\quad \lim_{n\to\infty} \frac{\ln n}{n} \to 0$

$\int_{x=1}^{\infty} \frac{\ln x}{x}\,dx \longrightarrow \lim_{b\to\infty} \int_1^b \frac{\ln x}{x}\,dx$

let $u = \ln x$

$du = \frac{dx}{x}$

$du = \frac{dx}{x}$

$x = 1 \Rightarrow u = \ln x = 0$

$x = b \Rightarrow u = \ln b$

$$\lim_{b\to\infty} \int_0^{\ln b} u\,du$$

$$= \lim_{b\to\infty} u^2/2 \Big|_{u=0}^{\ln b} = \infty$$

By integral Test, series diverges!

Final exam Review

① Evaluate

$$\int x^3 \sqrt{4-x^2}\,dx$$

TRIGNOMETRIC, U substitution!

$\sqrt{a^2 - x^2} \longrightarrow \sqrt{(\ )^2}$

→ Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $-\sin^2\theta$

$-\sin^2\theta$

$\boxed{\cos^2\theta = 1 - \sin^2\theta}$

Let $x = 2\sin\theta$

$\sqrt{4-x^2} = \sqrt{4-(2\sin\theta)^2}$

$= \sqrt{4 - 4\sin^2\theta}$

$= \sqrt{4(1-\sin^2\theta)} = \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

$dx = 2\cos\theta\,d\theta$

$\int x^3 \sqrt{4-x^2}\,dx$

$x = 2\sin\theta$ $\quad dx = 2\cos\theta\,d\theta$ & $\sqrt{4-x^2} = 2\cos\theta$

$$\int \underbrace{(2\sin\theta)^3}_{x^3} \underbrace{(2\cos\theta)}_{\sqrt{4-x^2}} \underbrace{(2\cos\theta\,d\theta)}_{dx}$$

$$8 * 2 * 2 \int \sin^3\theta \cos^2\theta\,d\theta = 32\int \sin^3\theta\cos^2\theta\,d\theta$$

Here $\sin\theta$ has an odd power,

$\sin^3\theta = \sin^2\theta \sin\theta$

$\sin^2\theta = \ldots$

$$\to 32\int \sin^2\theta \sin\theta \cos^2\theta \, d\theta$$

sin is odd power → $u = \cos\theta$

$$= 32 \int \sin^2\theta \cos^2\theta \, \sin\theta \, d\theta \quad du$$

$$(1-\cos^2\theta)\cos^2\theta \sin\theta \, d\theta$$

$$= 32 \int (1-\cos^2\theta)\cos^2\theta \sin\theta \, d\theta$$

Let $u = \cos\theta$ $\quad du = -\sin\theta \, d\theta$

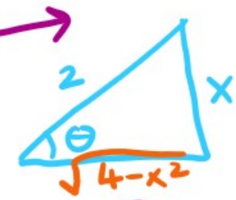
$$= 32 \int (1-u^2)u^2(-du)$$

$$= -32 \int (u^2 - u^4)\,du = -32\int u^2 du + 32\int u^4 du$$

$$= -32\, u^3/3 + 32\frac{u^5}{5} + c$$

$u = \cos\theta$

$x = 2\sin\theta$



$\sin\theta = \frac{\text{opp}}{\text{hypo}}$

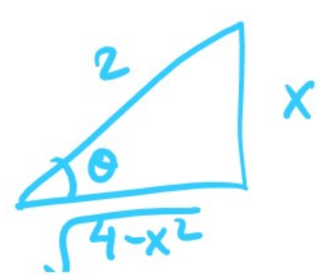
$\cos\theta = \frac{\text{Base}}{\text{hypo}}$

$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

$$= -32\frac{(\cos\theta)^3}{3} + \frac{32}{5}(\cos\theta)^5 + c$$

$$= \frac{-32}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3 + \frac{32}{5}\left(\frac{\sqrt{4-x^2}}{2}\right)^5 + c$$

$$\int x^3 \sqrt{4-x^2}\, dx$$



$$\sqrt{4-x^2}$$

② Find the limit $\lim_{x\to\infty} x^{1/x}$ (L'Hopital Rule) $\frac{\infty}{\infty}, \infty^0, \infty^\infty$

$$\lim_{x\to\infty} \frac{f'(x)}{g'(x)} = \lim_{x\to\infty} \frac{f(x)}{g(x)}$$

Let $y = \lim_{x\to\infty} x^{1/x}$ $\quad \ln y = \lim_{x\to\infty} \ln(x^{1/x})$

$$\ln y = \lim_{x\to\infty} \ln(x^{1/x}) = \lim_{x\to\infty} \frac{1}{x}\ln x \quad \frac{f(x)}{g(x)} \quad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{x^{-1}}{1} \quad \frac{f'(x)}{g'(x)}$$

$$\ln y = 0 \Rightarrow y = 1$$

$$\Rightarrow \boxed{\lim_{x\to\infty} x^{1/x} = 1}$$

④ $\int_1^\infty \frac{dx}{(3x+1)^3}$ evaluate the improper integral and check if it converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{(3n+1)^3}$$

$$\lim_{b\to\infty} \int_{x=1}^{b} \frac{dx}{(3x+1)^3}$$

$3x+1$

Let $u = 3x+1$
$du = 3dx \Rightarrow dx = \frac{1}{3}du$

$$\int \frac{dx}{(3x+1)^3} = \int \frac{du}{3u^3} = \frac{1}{3}\int \frac{du}{u^3} = \frac{1}{3}\frac{u^{-3+1}}{(-3+1)}$$
$$= -\frac{1}{6}u^{-2}$$
$$= -\frac{1}{6}(3x+1)^{-2}$$

$$\int_1^b \frac{dx}{(3x+1)^3} = -\frac{1}{6}(3x+1)^{-2}\Big|_{x=1}^{b}$$
$$= -\frac{1}{6}\left((3b+1)^{-2} - (3+1)^{-2}\right)$$

$$\lim_{b\to\infty}\int_1^b \frac{dx}{(3x+1)^3} = \lim_{b\to\infty}\left(-\frac{1}{6}\right)(3b+1)^{-2} + \frac{1}{6}4^{-2}$$

$$= \lim_{b\to\infty}\left(-\frac{1}{6}\right)\frac{1}{(3b+1)^2} + \frac{1}{6*4^2}$$

$$= \frac{1}{\infty} + \frac{1}{6*16} = 0 + \frac{1}{6*16}$$

$\int_1^\infty \frac{dx}{(3x+1)^3}$ converges.

#5 $\int (2x+1)e^{3x}dx = \int 2xe^{3x}dx + \int e^{3x}dx$

$\int 2xe^{3x}dx$ → integration by parts

$\int e^{3x}dx$ → $u = 3x$, $du = 3dx$, $\frac{du}{3} = dx$

by parts

$du = 3dx$

$\frac{du}{3} = dx$