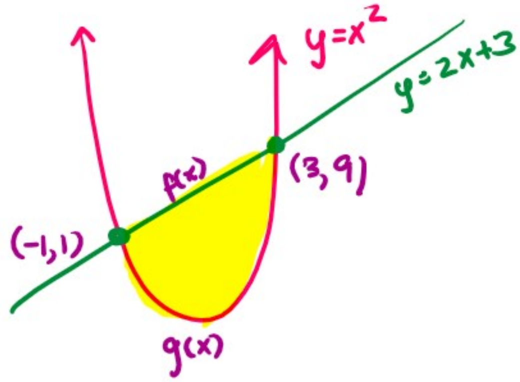
M1312

Thursday, November 21, 2019 8:50 AM

$y = x^2$

$y = 2x + 3$



Points of intersection:

$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$x = 3, \quad x = -1$$

Formula: $\left(\frac{M_y}{M}, \frac{M_x}{M}\right)$

$$M = \rho \int_{-1}^{3} (f(x) - g(x))\, dx = \rho \int_{-1}^{3} (2x + 3 - x^2)\, dx$$

$$= \rho\left(\cancel{2}\frac{x^2}{\cancel{2}} + 3x - \frac{x^3}{3}\right)\Big|_{-1}^{3}$$

$$= \rho(9 + 9 - 9) - \rho\left(\underbrace{1 - 3 + 1/3}_{\substack{-2 + 1/3 \\ -5/3}}\right)$$

$$= 9\rho - \left(-\frac{5}{3}\right)\rho$$

$$M = \left(\frac{27 + 5}{3}\right)\rho = \frac{32}{3}\rho$$

$$M_y = \rho \int_{-1}^{3} x(f(x) - g(x))\, dx$$

$$= \rho \int_{-1}^{3} x(2x + 3 - x^2)\, dx$$

$$= \int_{-1}^{3} (2x^2 + 3x - x^3)\, dx$$

$$= \rho \int_{-1}^{3} (2x^2 + 3x - x^3)\, dx$$

$$= \rho \left(\frac{2x^3}{3} + 3\frac{x^2}{2} - \frac{x^4}{4} \right) \Big|_{-1}^{3}$$

$$= \rho \left(\overset{18}{2*9} + \frac{3*9}{2} - \frac{81}{4} \right) - \rho \left(\frac{-2}{3} + \frac{3}{2} - \frac{1}{4} \right)$$

$$= \rho \left(\frac{36+27}{2} - \frac{81}{4} \right) - \rho \left(\frac{-2}{3} + \frac{6-1}{4} \right)$$

$$= \rho \left(\frac{63}{2} - \frac{81}{4} \right) - \rho \left(\frac{-8+15}{12} \right)$$

$$= \rho \left(\frac{126-81}{4} \right) - \rho \frac{7}{12}$$

$$= \rho \left(\frac{45}{4} - \frac{7}{12} \right)$$

$$= \rho \left(\frac{135-7}{12} \right)$$

$$= \frac{128}{12} \rho$$

$M_y = \frac{32}{3}\rho \longrightarrow$ same as M!

$\Rightarrow \quad \bar{x} = 1$

$$M_x = \rho \int_{-1}^{3} \left(\frac{f(x)+g(x)}{2}\right)\left(f(x)-g(x)\right) dx$$

$$= \frac{\rho}{2} \int_{-1}^{3} (f(x))^2 - (g(x))^2 \, dx$$

THIS WILL HELP!

$$= \frac{\rho}{2} \int_{-1}^{3} (2x+3)^2 - (x^2)^2 \, dx$$

$$\boxed{4x^2+12x+9-x^4} \quad dx$$

$$= \frac{\rho}{2} \left(\frac{4x^3}{3} + 6x^2 + 9x - \frac{x^5}{5}\right)\Bigg|_{-1}^{3}$$

$$= \rho\left(\frac{2x^3}{3} + 3x^2 + \frac{9}{2}x - \frac{x^5}{10}\right)\Bigg|_{-1}^{3}$$

$$M_x = \rho\left(\frac{2}{3}*27 + 27 + \frac{27}{2} - \frac{243}{10}\right)$$

$$- \rho\left(-\frac{2}{3} + 3 - \frac{9}{2} + \frac{1}{10}\right)$$

$$= \rho\left(\frac{2}{3}*28 + 24 + \frac{\cancel{36}}{\cancel{2}} - \frac{244}{10}\right)$$

56

18

$$= \rho\left(\frac{56}{3} + 42 - \frac{244}{10}\right)$$

$$= \rho\left(\frac{56}{3} + \frac{420-244}{10}\right) = \rho\left(\frac{56}{3} + \frac{88}{5}\right)$$

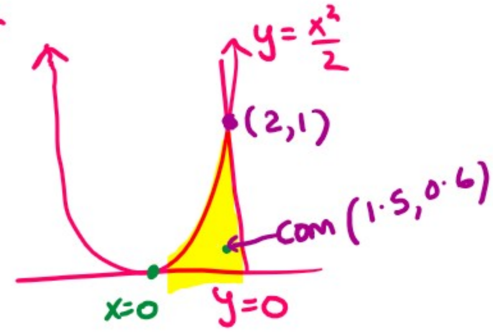
$$= \frac{544}{15}\rho$$

$$\bar{Y} = \frac{M_x}{M} = \frac{544/15}{32/3} = \frac{544}{15} * \frac{3}{32} = \frac{17}{5}$$

9

$y = x^2/2$ $\quad$ $y=0$ $\quad$ $x=2$

$f(x)$ $\quad$ $g(x)$



$$M = \rho\int_0^2 \frac{x^2}{2}\,dx$$

$$M = \frac{\rho}{2}\frac{x^3}{3}\Big|_0^2 = \frac{\rho}{2} * \frac{8}{3} = \frac{4}{3}\rho$$

$$M_y = \rho\int_0^2 x\left(\frac{x^2}{2} - 0\right)dx = \frac{\rho}{2}\frac{x^4}{4}\Big|_0^2 = \frac{\rho}{8} * 16 = 2\rho$$

$$M_x = \rho\int_0^2 \frac{1}{2}\left(x^2/2\right)^2 dx = \frac{\rho}{8}\int_0^2 x^4\,dx = \frac{\rho}{8} * \frac{32}{5} = \frac{4}{5}\rho$$

$$\bar{X} = 2\rho / \tfrac{4}{3}\rho = 2\rho * \frac{3}{4\rho} = \frac{3}{2} = 1.5$$

$$\bar{Y} = \tfrac{4}{5}\rho / \tfrac{4}{3}\rho = \frac{4}{5}\rho * \frac{3}{4\rho} = 3/5 = 0.6$$

$\boxed{\text{chapter 9}}$ only #1, #2, #3 & #4

Chapter 9 | Only #1, #2, #3 & #4

① $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots$

formula for $a_n = ?$ $\quad a_n = \frac{(-1)^{n+1}}{(n+1)} n^2, \quad n = 1, 2, \ldots$

② exact sum for $\sum_{n=3}^{\infty} \frac{2^n}{5^{n+1}}$

Notice: $n = 3, 4, \ldots$

$\frac{2^3}{5^4} + \frac{2^4}{5^5} + \frac{2^5}{5^6} + \frac{2^6}{5^7} + \frac{2^7}{5^8} + \cdots$

$a + ar + ar^2 + \ldots$

$a = 2^3/5^4 \qquad \frac{a}{1-r}$

$r = 2/5$

③ Test for Convergence and clearly state which test you are using!

a) $\sum_{n=1}^{\infty} \frac{n+1}{n^2+2}$ $\quad \lim_{n \to \infty} \frac{n+1}{n^2+2} = \lim_{n \to \infty} \frac{(n+1)/n^2}{\frac{n^2}{n^2} + \frac{2}{n^2}} = \frac{1/n + 1/n^2}{1 + 2/n^2}$

LIMIT COMPARISON! $\qquad = 0$

$\frac{a_n}{b_n} = \frac{n+1}{n^2+2} * \frac{n^2}{n} = \frac{(n+1)/n}{\frac{n^2}{n^2} + 2/n^2} = \frac{1 + 1/n}{1 + 2/n^2} \to 1$ as $n \to \infty$

b) $\sum_{n=0}^{\infty} \frac{3^n}{n^3+3}$

Divergence Test?

Limit Comparison Test $\lim \frac{a_n}{b_n} = L > 0$

$a_n = \frac{3^n}{n^3+3}$

Σb_n and Σa_n same behavior.

$\frac{a_n}{b_n} = \frac{\cancel{3^n}}{n^3 + 3/n^3} * \boxed{\frac{\cancel{n^3}}{\cancel{3^n}}} = \frac{1}{1 + 3/n^3}$

∞

b_n $\quad n^3 + 3/n^3 \quad \cancel{\frac{3^n}{3^n}} \quad 1 + 3/n^3$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n)!\,(n-1)!}$

$\displaystyle\lim \frac{a_n}{b_n} = 1$

$\displaystyle\frac{1}{b_n} = \frac{n^3}{3^n} \Rightarrow b_n = \frac{3^n}{n^3}$

$3^n/n^3$

$$\lim_{n\to\infty} \frac{3^n}{n^3} = \lim_{x\to\infty} \frac{3^x}{x^3} = \lim_{x\to\infty} \frac{3^x \ln 3}{3x^2}$$

$$= \lim_{x\to\infty} \frac{(3^x \ln 3)\ln 3}{6x}$$

$$= \lim_{x\to\infty} \frac{(3^x \ln 3)(\ln 3)^2}{6}$$

$= \infty$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^2+n+1}$

Convergent

$a_{n+1} < a_n = \dfrac{1}{2n^2+n+1}$

$= 2(n+1)^2 + (n+1) + 1$

$\displaystyle\lim_{n\to\infty} \frac{1}{2n^2+n+1} = 0$

convergent

e) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{e^n}$

$-\frac{1}{e^1} + \frac{1}{e^2} - \frac{1}{e^3} + \dots$

$$\boxed{(f) \sum_{n=1}^{\infty} \frac{n+1}{n}}$$

$-1/e + 1/e^2 - 1/e^3 + \dots \quad a_{n+1} < a_n \quad a_n = \frac{1}{e^n}$

$\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

#4 Determine if the series

#4 Determine if the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3n}$$ is divergent
absolutely convergent
conditionally convergent.

$$\sum_{n=1}^{\infty} \frac{n+1}{n} \longrightarrow a_n = \frac{n+1}{n}$$

Limit Comparison Test

$$\frac{a_n}{b_n} = \frac{n+1}{n} * \frac{n}{n} = 1 + 1/n$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 1 \qquad \sum b_n = \sum 1$$

$$\sum_{n=1}^{\infty} \frac{n+1}{n}$$

n^{th} Term Divergence test

$\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum a_n$ diverges

$$a_n = \frac{n+1}{n} = \frac{n}{n} + \frac{1}{n} = 1 + \frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (1 + 1/n) = 1$$

DIVERGES

Divergence Test!

$$\sum^{\infty} \frac{\ln n}{} \qquad a_n = \frac{\ln n}{n}$$

$$\sum_{n=1}' \frac{\ln n}{n} \qquad a_n = \frac{\ln n}{n}$$

$$\lim_{n\to\infty} \frac{\ln n}{n} = \lim_{x\to\infty} \frac{\ln x}{x} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty} \frac{x^{-1}}{1} = 0$$

$$\sum_{n=1}^{\infty} \frac{\ln n}{n} \qquad \frac{\ln 1}{1} + \frac{\ln 2}{2} + \frac{\ln 3}{3} + \cdots$$

Geom. series X

Not a p series X

Divergence Test X

$\boxed{\text{Integral Test}}$

$$\int_{x=1}^{\infty} \frac{\ln x}{x}\, dx$$

$$= \lim_{b\to\infty} \int_{x=1}^{b} \frac{\ln x}{x}\, dx$$

Let $u = \ln x$

$du = \frac{dx}{x}$

$x=1 \Rightarrow u = \ln 1 = 0$

$x=b \Rightarrow u = \ln b$

$$= \lim_{b\to\infty} \int_0^{\ln b} u\, du = \lim_{b\to\infty} \frac{u^2}{2} \Bigg|_{u=0}^{\ln b} = \infty$$

$\Rightarrow$ By integral Test

∞

$\Rightarrow$ By integral test

$$\sum_{n=1}^{\infty} \frac{\ln n}{n} \text{ diverges.}$$

(c) $$a_n = \frac{n!}{(2n)!\,(n-1)!} \qquad n \geq 1$$

$$\left.\begin{aligned} n! &= 1 * 2 * \ldots n \\ (n-1)! &= 1 * 2 * \ldots (n-1) \end{aligned}\right\} \Rightarrow \frac{n!}{(n-1)!} = n$$

$$\boxed{a_n = \frac{\cancel{n!}}{(\cancel{n-1})!} \frac{1}{(2n)!} = \frac{n}{(2n)!}}$$

Factorial involved $\Rightarrow$ Apply Ratio Test.

$$\frac{|a_{n+1}|}{|a_n|} =$$

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{(2n+2)!} * \frac{(2n)!}{n}$$

$$(2n+2)! = (2n+2) * (2n+1) * 2n * (2n-1) * \ldots 1$$

$$(2n)! = 2n * (2n-1) * \ldots 1$$

$$\frac{(2n)!}{(2n+2)!} = \frac{\overbrace{\cancel{2n * (2n-1) * \ldots 1}}^{(2n)!}}{(2n+2)(2n+1) * \underbrace{\cancel{2n * \ldots 1}}_{(2n)!}} = \frac{1}{(2n+2)(2n+1)}$$

$(2n+2)!$ $\quad (2n+2)(2n+1) * \underbrace{\cancel{(2n)\ldots}}_{(2n)!}$

$(2n)! = 2n * 2n-1 * 2n-2 \ldots * 1$

$N = n+1$

$(2N)! = (2n+2)!$

$a_n = 2n \longrightarrow$ $a_1 = 2$, $a_2 = 4$, $a_3 = 6$ $\quad n=3 \quad 2n=6$

$n=2$

$4!$

$n=2 \quad (2n)! = (2*2)! = 4! = 4*3*2*1$

$2n \qquad 2, 4, 6, 8, 10, 12 \ldots$

$(2n+1)!$

$3, 5, 7, \ldots$

$$a_n = \frac{(n)!}{(2n)!(n-1)!} = \frac{n}{(2n)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{n+1}}{\cancel{(2(n+1))!}} * \frac{\cancel{(2n)}!}{\cancel{n}} = \frac{(1+1/n)}{(2n+2)(2n+1)}$$

$(2n+2)(2n+1)$

$a_n \qquad \cancel{(4(n+1))}! \qquad \cancel{n} \qquad (2n+2)(2n+1)$

$(2n+2)(2n+1)$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1+0}{\infty} = 0 < 1$$

$\Rightarrow$ Ratio Test $\sum \frac{n!}{(2n)!\,(n-1)!}$ converges.

$$\sum_{n=1}^{\infty} \frac{n}{n^2+2}$$

$$\frac{a_n}{b_n} = \frac{n+1}{n^2+2} * \frac{n^2}{n} = \frac{(n+1)/n}{(n^2+2)/n^2}$$

$$= \frac{(1+1/n)}{1+2/n^2}$$

$\lim \frac{a_n}{b_n} = 1 > 0 \Rightarrow \sum a_n$ & $\sum b_n$ have same behavior

$b_n = ?$ $\quad \frac{1}{b_n} = \frac{n^2}{n} \Rightarrow b_n = 1/n$

$\sum b_n$? $\quad \sum 1/n$ Diverges $\Rightarrow \sum a_n$ also diverges.