Series

Thursday, November 14, 2019 3:02 PM

(1) $\leftarrow$ $1, \ 1/2, \ 1/2^2, \ 1/2^3, \ 1/2^4, \ 1/2^5, \ 1/2^6, \ldots$

(2) $\leftarrow$ $\frac{1}{1}, \ 1/2^2, \ 1/3^2, \ 1/4^2, \ 1/5^2, \ 1/6^2, \ldots$

$a_n = ?$ for (1) $a_n = 1/2^n$ $n = 0, 1, 2, \ldots$

$2^0 = 1$

$a_n = ?$ for (2)

(2) $a_n = 1/n^2$

Diff. bet. a seq & a series

Sequences : $5, \ 5/6, \ 5/6^2, \ 5/6^3, \ldots$

$5 + 5/6 + 5/6^2 + 5/6^3 + \ldots$

Series $5 + 5/6 + 5/6^2 + 5/6^3 + \ldots$

$\boxed{a + ar + ar^2 + \ldots}$ if $|r| < 1$ then

$a/1-r$

first term

$5 + 5/6 + 5/6^2 + 5/6^3 + \ldots$

$a = 5$ $\quad r = \frac{5/6}{5} = 1/6 < 1$

So series converges to $\frac{5}{1 - 1/6} = 6$

Integral Test : evaluate

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Pre Requisite $\rightarrow$ $\displaystyle\int_1^{\infty} \frac{2x}{x^2+1}\,dx$ and check if it converges or not.

$$\int_1^{\infty} \frac{2x}{x^2+1}\,dx = \lim_{b\to\infty} \int_1^{b} \frac{2x}{x^2+1}\,dx$$

Let $x^2+1 = u$ $\rightarrow$ $x = 1 \Rightarrow u = 2$

$du = 2x\,dx$ $\quad x = b \Rightarrow u = b^2+1$

$$= \lim_{b\to\infty} \int_{u=2}^{b^2+1} \frac{du}{u}$$

$$= \lim_{b\to\infty} \ln|u| \Big|_{u=2}^{b^2+1}$$

$$= \lim_{b\to\infty} \ln|b^2+1| - \ln 2$$

$= \infty$ Divergence of $\displaystyle\int_1^{\infty} \frac{2x\,dx}{x^2+1}$

Integral Test

$\displaystyle\sum_{n=1}^{\infty} a_n$ converges if

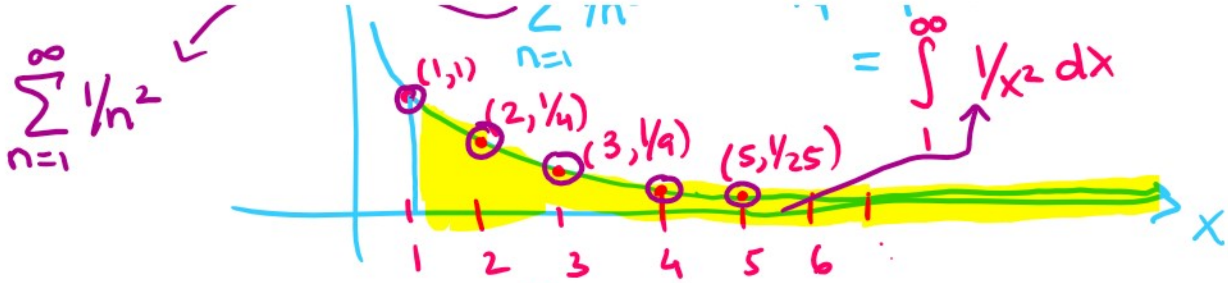
I can find $f(x)$ $\quad a_n = f(n) \quad n = 1, 2, 3, 4, \ldots$

such $\displaystyle\sum_{n=1}^{\infty} a_n = \int_1^{\infty} f(x)\,dx$

$\displaystyle\sum_{n=1}^{\infty} 1/n^2 \quad 1 + 1/4 + 1/9 + 1/16 + 1/25 + \ldots$

$= \displaystyle\int^{\infty} 1/x^2\,dx$

$\displaystyle\sum^{\infty} 1/n^2$



Check the series $\sum_{n=1}^{\infty} 1/n^2$ for convergence.

$\lim_{n\to\infty} 1/n^2 = 0$ need further testing!

$$\sum_{n=1}^{\infty} \longrightarrow \int_{x=1}^{\infty}$$

$$1/n^2 \longrightarrow 1/x^2\,dx$$

$$\sum_{n=1}^{\infty} 1/n^2 = \int_1^{\infty} 1/x^2\,dx$$

$$= \lim_{b\to\infty} \int_1^b 1/x^2\,dx$$

$$= \lim_{b\to\infty} \frac{x^{-2+1}}{-2+1}\Bigg|_{x=1}^{b}$$

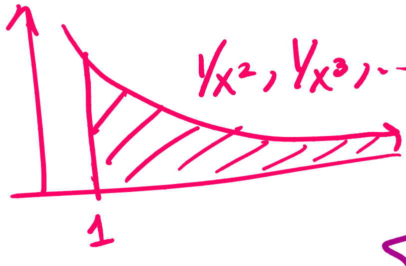
$$= \lim_{b\to\infty} -\left(b^{-1} - 1\right).$$

$$\int_1^{\infty} \frac{dx}{x^2} = 1 \quad \text{Convergence!}$$

$$\Rightarrow \sum_{n=1}^{\infty} 1/n^2 = 1$$

It turns out: $\sum^{\infty} 1/n^p$ $p>1$ then convergence

It turns out: $\sum_{n=1} 1/n^p$ $\quad p>1$ then Convergence



$1/x^2, 1/x^3, \ldots$ all converge!

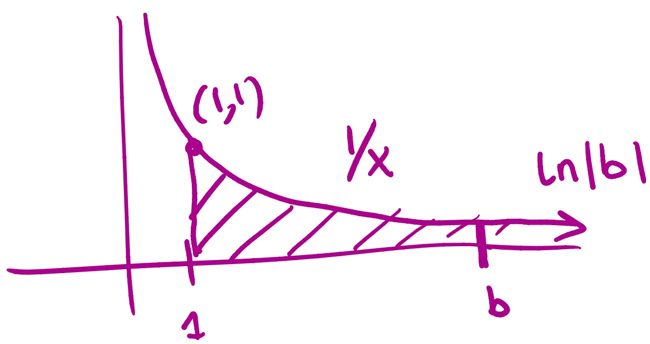
Counter Intuitive example $\sum_{n=1}^{\infty} 1/n = \int_{x=1}^{\infty} 1/x \, dx$

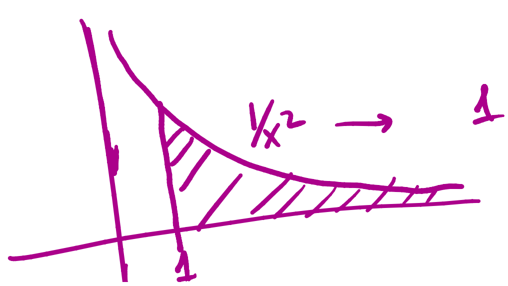
$$\lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx = \lim_{b\to\infty} \ln|x| \Big|_{x=1}^{b}$$

$$= \lim_{b\to\infty} \ln|b| - \ln 1$$

$$= \lim_{b\to\infty} \ln|b| = \infty$$

$\Rightarrow \int_1^{\infty} 1/x \, dx$ diverges so $\sum_{n=1}^{\infty} 1/n$ diverges





check for conv.

$$\sum_{n=2}^{\infty} \frac{\ln n}{n} \qquad \text{Typically } \frac{n^2}{n^3+1}, \ldots$$

tests other than Integral test.

log fuction

$$\sum_{n=2}^{\infty} \frac{\ln n}{n} = \int_{x=2}^{\infty} \frac{\ln x}{x}\, dx = \lim_{b\to\infty} \int_{x=2}^{b} \frac{\ln x}{x}\, dx$$

Let $u = \ln x$

$du = \frac{dx}{x}$

$x=2 \Rightarrow u = \ln 2$ and $x = b$ $u = \ln b$

$$= \lim_{b\to\infty} \int_{u=\ln 2}^{\ln b} u \, dx$$

$$= \lim_{b\to\infty} \frac{1}{2} u^2 \Big|_{\ln 2}^{\ln b}$$

$$= \lim_{b\to\infty} \frac{1}{2}(\ln b)^2 - \frac{1}{2}(\ln 2)^2$$

$$= \infty$$

$$\sum_{n=1}^{\infty} 1/n^{10} \qquad \lim_{n\to\infty} 1/n^{10} = 0$$

conclusion

$n=1$ $\quad n \to \infty$

$\Rightarrow$ no conclusion

$$\int_{x=1}^{\infty} \frac{dx}{x^{10}} = \lim_{b\to\infty} \left. \frac{x^{-10+1}}{-9} \right|_{x=1}^{b}$$

$$= \lim_{b\to\infty} \frac{b^{-9}}{-9} - \frac{1}{-9} = 1/9$$

Test the series $\sum_{n=1}^{\infty} 1/\sqrt{n}$ for convergence

$$\int_{x=1}^{\infty} 1/\sqrt{x}\, dx$$

$$= \lim_{b\to\infty} \int_1^b 1/\sqrt{x}\, dx$$

$$= \lim_{b\to\infty} \int_1^b x^{-1/2}\, dx$$

$$= \lim_{b\to\infty} \left. \frac{x^{-1/2+1}}{-1/2+1} \right|_{x=1}^{b}$$

$$= \lim_{b\to\infty} \frac{b^{1/2}}{1/2} - \frac{1}{1/2}$$

$= \infty$

$\Rightarrow \sum_{n=1}^{\infty} 1/n^{1/2}$ diverges by integral test!

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series

$$\sum_{n=1}^{\infty} 1/n^p$$ p-series

$p \leq 1$

$\Rightarrow$ divergence

$p > 1 \Rightarrow$ convergence

Comparison Test

Basic Comparison Test

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1} \Rightarrow \int_1^{\infty} \frac{dx}{x^2+1} = \lim_{b \to \infty} \arctan x \Big|_1^b$$

$\frac{1}{n^2+1}$ can be bounded from above by a convergent series, then $\sum 1/n^2+1$ converges

$$1/n^2+1 < 1/n^2$$

$\sum 1/n^2$ Converges $p = 2 > 1$

Smaller series $\leq$ Convergent Bigger series

$\Rightarrow$ smaller series converges!

$\Rightarrow$ smaller series converges.

$\Rightarrow$ $\sum 1/n^2+1$ converges by Basic Comp. Test

$$\sum_{n=2}^{\infty} 1/n-1 = ?$$

$$\int_2^{\infty} \frac{dx}{x-1} = \lim_{b\to\infty} \int_2^b \frac{dx}{x-1}$$

$\Rightarrow \infty$

$n \geq 2,$

$$n-1 < n$$

$$1/n-1 > 1/n$$

known Divergent $\sum 1/n$

$\sum 1/n-1$ has same behavior as $\sum 1/n$

$\sum 1/n-1$ also diverges.

$$1/n^2+1 < 1/n^2$$

$$1/n < 1/n-1, \; n \geq 2$$