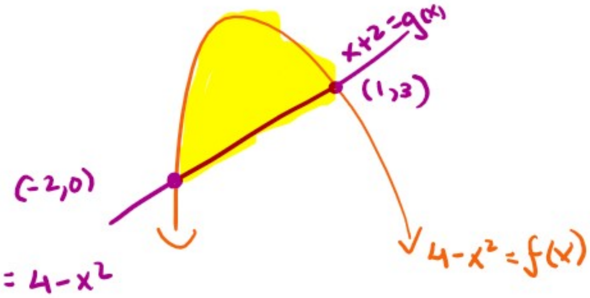
CoM for Laminar

Thursday, November 7, 2019 3:08 PM

Centroid
CoM of region
bounded
by



$f(x) = 4-x^2$

$g(x) = x+2$

$$CoM = (\bar{x}, \bar{y})$$

$$= \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

$M_y \to$ moments about y-axis

$\sum m_i x_i \to \rho * \text{Area}$

$M_x \to$ moments about x-axis

$\sum m_i y_i$

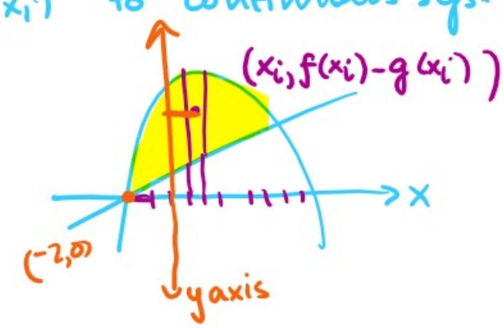
$m = \rho * \text{Area}$, ρ = Density

Going from (m_i, x_i) to continuous sys.

$\sum_{i=1}^{n} m_i x_i \to$

$m_i = \rho * \text{Area}$

$= \rho * (f(x_i) - g(x_i))$



x_i = distance from y-axis

$$\frac{M_y}{M} = \frac{\int_a^b \cancel{\rho}\,(f(x)-g(x))\,x\,dx}{\int_a^b \cancel{\rho}\,(f(x)-g(x))\,dx}$$

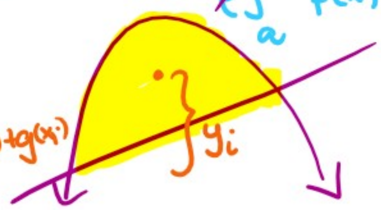
$$\bar{x} = M_y/m = \frac{\int_a^b (f(x)-g(x))\,x\,dx}{\int_a^b f(x)-g(x)\,dx}$$

$\int^b \cancel{\rho}\,(f(x)-g(x))\ \tfrac{1}{2}(f(x)+g(x))\ dx$

$$\bar{y} = \frac{M_x}{M} = \frac{\int_a^b \cancel{\rho}(f(x)-g(x))\, \frac{1}{2}(f(x)+g(x))\, dx}{\cancel{\rho}\int_a^b f(x)-g(x)\, dx}$$

$$M_x = \sum_{i=1}^{n} m_i y_i \rightarrow$$

$$y_i = \frac{1}{2} f(x_i)+g(x_i)$$



$$(\bar{X}, \bar{Y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$$

$$M_y = \rho \int_a^b \underbrace{x}_{x_i} \underbrace{(f(x) - g(x))}_{m_i}\, dx$$

$$M_x = \rho \underbrace{\int_a^b}_{\sum_{i=1}^{n}} \underbrace{(f(x)-g(x))}_{m_i} \underbrace{\left(\frac{f(x)+g(x)}{2}\right)}_{y_i} dx$$

$f(x) = 4-x^2$ $\quad g(x) = x+2$

$a = -2$ $\quad b = 1$

$$M_y = \rho \int_{-2}^{1} (f(x)-g(x))\, x \, dx$$

$$= \rho \int_{-2}^{1} (4-x^2-x-2)\, x \, dx$$

$$= \rho \int_{-2}^{1} (4x - x^3 - x^2 - 2x)\, dx$$

$$2x - x^3 - x^2$$

$$= \rho \int_{-2}^{1} (2x - x^3 - x^2)\, dx$$

$$M_y = \rho \left(x^2 - \frac{x^4}{4} - \frac{x^3}{3} \right) \Big|_{x=-2}^{1}$$

$$M_y = \rho\left(x^2 - \frac{x^4}{4} - \frac{x^3}{3}\right)\Big|_{x=-2}^{1}$$

$$= \rho\left(\underbrace{1 - 1/4 - 1/3}_{\substack{1 - 7/12 \\ 5/12}}\right) - \rho\underbrace{\left(4 - \frac{\cancel{16}^{\,4}}{\cancel{4}} - \frac{(-8)}{3}\right)}_{8/3}$$

$$M_y = \rho\left(\frac{5}{12} - \frac{8}{3}\right) = \rho\left(\frac{5-32}{12}\right) = -\frac{27}{12}\rho$$

$$= -9/4\,\rho$$

$$m \longrightarrow \Sigma m_i$$

$$\hookrightarrow m = \rho \int_{-2}^{1} f(x) - g(x)\,dx$$

$$= \rho \int_{-2}^{1} (\underbrace{4 - x^2 - x - 2}_{2-x^2-x})\,dx$$

$$= \rho \int_{-2}^{1} 2 - x^2 - x \quad dx$$

$$= \rho\left(2x - \frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_{-2}^{1}$$

$$= \rho\left(2 - 1/3 - 1/2\right) - \rho\left(\underbrace{-4 + 8/3 - \cancel{4}/\cancel{2}^{\,2}}_{-4 + (-2) + 8/3 = 2/3 - 4}\right)$$

$$2 - 5/6$$

$$\frac{12-5}{6} = 7/6$$

$$= -10/3$$

$$= \rho\left(7/6\right) - \rho\left(-10/3\right)$$

$$= \rho\left(\tfrac{7}{6}\right) - \rho\left(-\tfrac{10}{3}\right) \qquad = -10/3$$

$$= \tfrac{7}{6}\rho + \tfrac{10}{3}\rho = \left(\tfrac{7}{6} + \tfrac{20}{6}\right)\rho = \tfrac{27}{6}\rho$$

$$\boxed{m = 9/2\,\rho}$$

$$\bar{x} = m_y / m$$

$$= -9/4\,\cancel{\rho} \,/\, 9/2\,\cancel{\rho} = -9/4 \div 9/2 = -\tfrac{9}{4} \times \tfrac{2}{9} = -\tfrac{1}{2}$$

$$\bar{y} = m_x / m \qquad \sum m_i y_i \Big/ \sum m_i$$

$$m = 9/2\,\rho$$

$$m_x = \rho \int_{-2}^{1} \left(\frac{f(x)+g(x)}{2}\right)\left(f(x)-g(x)\right)dx$$

$$\frac{(A+B)}{2}(A-B) = \frac{A^2-B^2}{2}$$

$$= \frac{\rho}{2}\int_{-2}^{1}\left(f(x)^2 - g(x)^2\right)dx$$

$$m_x = \frac{\rho}{2}\int_{-2}^{1} (4-x^2)^2 - (\cancel{x}+2)^2\,dx$$

$$(a-b)^2 = a^2 - 2ab + b^2 \quad (a = 4,\ b = x^2)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \frac{\rho}{2}\int_{-2}^{1}\left(16 - 8x^2 + x^4\right) - \left(x^2 + 4x + 4\right)dx \quad (a^2,\ 2ab,\ b^2)$$

$$12 - 9x^2 - 4x + x^4$$

$$= \frac{\rho}{2}\int_{-2}^{1}\left(12-9x^2-4x+x^4\right)dx$$

$$= \frac{\rho}{2}\left(12x-\frac{\cancel{9}^{\,3}x^3}{\cancel{3}}-\frac{\cancel{4}^{\,2}x^2}{\cancel{2}}+\frac{x^5}{5}\right)_{-2}^{1}$$

$x=-2$

$$=\frac{\rho}{2}\Big(\underbrace{12-3-2}_{7}+1/5\Big)-\frac{\rho}{2}\left(-\cancel{24}+\cancel{24}-2*4-\frac{32}{5}\right)$$

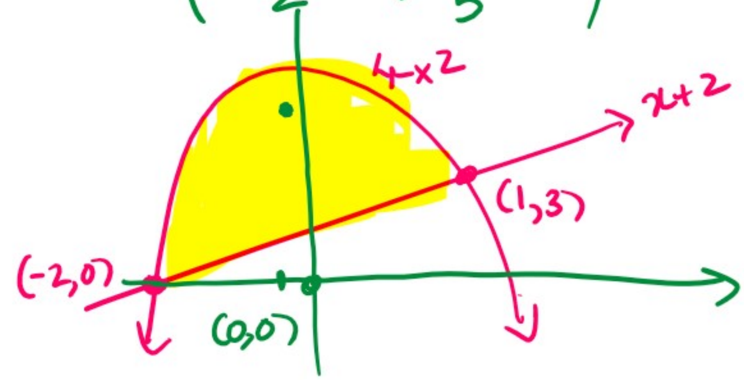
$7+1/5$ $\qquad$ $-8-32/5$

$36/5$ $\qquad$ $-72/5$

$$m_x=\frac{\rho}{2}\left(\frac{36}{5}-\left(-\frac{72}{5}\right)\right)=\frac{108}{5}\rho/2=\frac{54\rho}{5}$$

$$\bar{y}=\frac{54/5\,\rho}{9/2\,\rho}=\frac{\cancel{54}^{\,6}}{5}*\frac{2}{\cancel{9}}=\frac{12}{5}$$

final Answer: $\left(-\frac{1}{2},\frac{12}{5}\right)$



<u>chapter 9</u> : 10% integrals

a_1 $\quad$ Discrete points

a_2 $\quad$ growth of bacteria recorded

a_1 Discrete points

a_2 growth of bacteria recorded

a_3 over 1 min

a_4 2 min

a_5 3 min

a_6 ⋮

⋮

Coding

$$\overset{a_1}{1/2},\ \overset{a_2}{1/4},\ \overset{a_3}{1/8},\ 1/16,\ 1/32,\ \ldots\ 1/2^n$$

$$a_n = 1/2^n$$

$n=3 \qquad a_3 = 1/8$

Predictive modeling $\quad a_{1000} = \dfrac{1}{2^{1000}}$

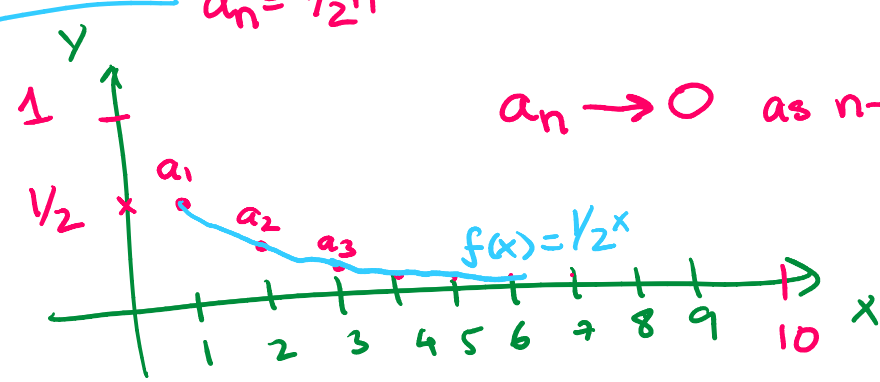
$$\lim_{n\to\infty} a_n = \ ?$$

Techniques to calculate this limit.

(Pre Requisite : L'Hopital Rule)

$a_n = 1/2^n$

$a_n \to 0$ as $n \to \infty$



$a_n = 1/2^n$

$\rightarrow a_n = 1/2^n$

Define $f(x) = 1/2^x$

$$\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x)$$

To calculate $\lim_{n\to\infty} a_n$, introduce $f(x)$ where

$$f(n) = a_n \qquad n = 1, 2, 3, \dots$$

e.g: $f(x) = 1/2^x \rightarrow$ $f(1) = 1/2 = a_1$

$f(2) = 1/2^2 = a_2$

$f(3) = 1/2^3 = a_3$

example: $a_n = \dfrac{n^2}{2^n - 1}$

Calculate first 5 terms of $\{a_n\}$

$\hookrightarrow$ SEQUENCE a_n

$a_1 = 1/(2-1) = 1$ $\qquad a_3 = 3^2/(2^3-1) = 9/7$

$a_2 = 2^2/(2^2-1) = 4/3$ $\qquad a_4 = 4^2/(2^4-1) = 16/15 = 1.0667$

$$a_5 = \frac{5^2}{2^5-1} = \frac{25}{32} = 0.78125$$

$$a_6 = \frac{6^2}{2^6-1} = \frac{36}{63} = 0.5714$$

converging to 0 ?

$$\lim a_n \overset{?}{=} \lim \frac{n^2}{2^n}$$

$$\lim_{n\to\infty} a_n \stackrel{?}{=} \lim_{n\to\infty} \frac{n^2}{2^n-1}$$

Replace n with x

$$\lim_{n\to\infty} \frac{n^2}{2^n-1} = \lim_{x\to\infty} \frac{x^2}{2^x-1} \quad \frac{\infty}{\infty} \quad f/g$$

L'Hopital Rule

$$= \lim_{x\to\infty} \frac{2x}{2^x \log 2} \quad \left(\frac{\infty}{\infty}\right)$$

L'Hopital Rule

$$= \lim_{x\to\infty} \frac{2}{(2^x \log 2)^2}$$

$$= \frac{2}{\infty} = 0$$

Given sequence

$$\frac{2}{1}, \frac{4}{3}, \frac{8}{5}, \underset{n=4}{\frac{16}{7}}, \underset{n=5}{\frac{32}{9}}, \dots$$

$$\boxed{a_n = \frac{2^n}{2n-1}}$$

$n=1 \to 1$

$n=2 \to 3$

$n=3 \to 5$

$n=4 \to 7$

$n=1 \to 2*n-1 = 1$ ✓

$n=2 \to 2*n-1 = 3$ ✓

$n=4 \to 2*4-1 = 7$ ✓

$$\lim_{n\to\infty} 1/n!$$

$n!$ = factorial of n

= Product of first n positive integers

$3! = 1*2*3 = 6$

$3! = 1*2*3 = 6$

$5! = 5*4*3*2*1$

$0! = \underline{1}$

$\lim_{n\to\infty} \frac{1}{n!} = \lim_{x\to\infty} \frac{1}{x!}$ not applicable

→ Squeeze Theorem

→ Bounded, monotonic sequences.