2 problems →

Tuesday, November 5, 2019 5:00 PM

Hooke's Law
area of region bdd } 40 pts

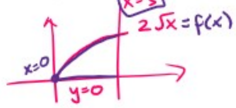
#2 $y = 2\sqrt{x}$

$y = 0$ $x = 3$

revolved about x-axis.

Find $\boxed{\text{surface area}}$ of solid generated

$$S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\, dx$$



$y = 2\sqrt{x}$ (TOP)

$y = 0$ (BOTTOM)

$$f'(x) = 2\frac{d}{dx}(\sqrt{x}) = 2\ \frac{1}{2}x^{-1/2} = \frac{1}{\sqrt{x}}$$

$$f'(x)^2 = \frac{1}{x}$$

$$\sqrt{1 + f'(x)^2} = \sqrt{1 + 1/x} = \sqrt{\frac{x+1}{x}}$$

$$= \frac{\sqrt{x+1}}{\sqrt{x}}$$

$$S = 2\pi \int_0^3 2\cancel{\sqrt{x}}\sqrt{\frac{x+1}{\cancel{x}}}\ dx$$

$$= 4\pi \int_{x=0}^{3} \sqrt{x+1}\ dx \qquad u = x+1,\ du = dx$$

$$= 4\pi \int_{u=1}^{4} \sqrt{u}\ du$$

$$= 4\pi\ \frac{u^{1/2+1}}{(1/2+1)} \Big|_{u=1}^{4}$$

$$= 4\pi * \frac{1}{3/2}\ u^{3/2} \Big|_{u=1}^{4}$$

$$= \frac{8\pi}{3}\left(4^{3/2} - 1\right)$$

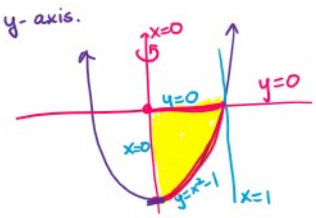
$$4^{3/2} = (2^2)^{3/2} = 2^3 = 8$$

$$= \frac{56\pi}{3}$$

4 $y = x^2 - 1$ $y = 0$

$x = 0$ $x = 1$

about y-axis.



Look Below.

#6 ... $f(x) = (x^2+2)^{3/2}$ over $[-1, 2]$

#5 Option 2: $f(x) = \frac{(x^2+2)^{3/2}}{3}$ over $[-1, 2]$

$$s = \int_{-1}^{2} \sqrt{1+f'(x)^2}\, dx$$

$$f(x) = \frac{1}{3}(x^2+2)^{3/2} \rightarrow f'(x) = \frac{1}{3} * \frac{3}{2}(x^2+2)^{3/2-1}(2x)$$

$$f'(x) = \frac{1}{\cancel{3}} * \frac{\cancel{3}}{\cancel{2}}(x^2+2)^{1/2}(\cancel{2}x)$$

$$f'(x)^2 = \left((x^2+2)^{1/2} x\right)^2$$

$$= x^2(x^2+2)$$

$$= x^4 + 2x^2$$

$$\sqrt{1+f'(x)^2} = \sqrt{1+x^4+2x^2} \rightarrow (a^2+b^2+2ab)$$

$$= \sqrt{(1+x^2)^2}$$

$$= (1+x^2)$$

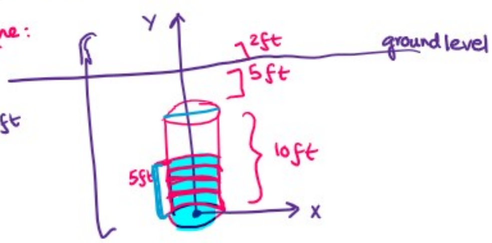
$$s = \int_{-1}^{2} (1+x^2)\, dx = \left(x + \frac{x^3}{3}\right)\Big|_{x=-1}^{2}$$

$$= (2 + 8/3) - (-1 - 1/3)$$

$$= 2 + 8/3 + 1/3 + 1$$

$$= 3 + 3 = 6$$

Work Done:



Total height = 17ft

force → volume of fluid

$50\,(\pi * r^2\, \Delta y)$ $(17-y)$

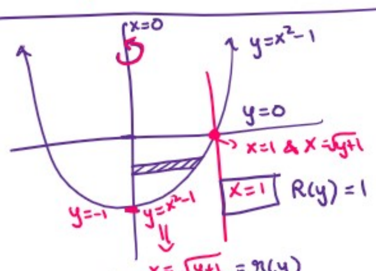
$4\pi \Delta y$ ↳ sum formula $y=0 \rightarrow y=5$

$$W = \int_{y=0}^{5} 50 * \pi * 2^2\, (17-y)\, dy$$

$$= 200\pi \int_0^5 (17-y)\, dy$$

$$= 200\pi \left(17*5 - \frac{25}{2}\right)$$

#2



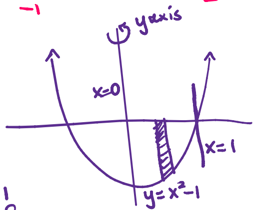
Washer

$$V = \pi \int_c^d \left(R(y)^2 - r(y)^2\right) dy$$

$$= \pi \int_{-1}^{0} 1^2 - \left(\sqrt{1+y}\right)^2 dy$$

$$= \pi \int_{-1}^{0} (-y)\,dy = \frac{\pi}{2}$$

$\boxed{\text{Shell}}$



$$V = 2\pi \int_{x=0}^{1} x\left(0-(x^2-1)\right)dx = 2\pi \int_{0}^{1} (x-x^3)\,dx$$

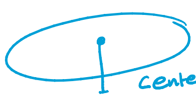
$$= 2\pi\left(\frac{1}{2}-\frac{1}{4}\right) = \pi/2$$

Center of Mass: (7.6)

$\boxed{\text{mass}}$: body's resistance to changes in motion indep. of gravational sys.

$m_1x_1 = m_2x_2$

Location of "center" about which $m_1x_1 = m_2x_2$

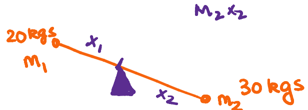


Center of Mass
Location of position

moment about a point — mass * distance from point

m_1x_1

m_2x_2



find x_1 and x_2 so that $m_1x_1 = m_2x_2$.

m_1 at $x_1 = 10$, $x_2 = 10$ } from ▲

moment of $m_1 = 20*10 = 200$

moment of $m_2 = 30*10 = 300$

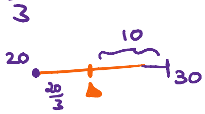
can we find x_1 and x_2 so that $m_1x_1 = m_2x_2$

$$20 * x_1 = 30 * x_2$$

$$\frac{20}{30} = \frac{x_1}{x_2}$$

$$\frac{x_1}{x_2} = \frac{20}{30} = \frac{2}{3}$$

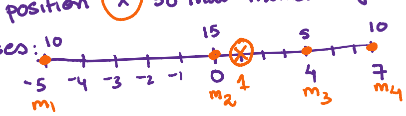
$$\boxed{x_1 = 2/3\, x_2}$$



$x_2 = 10$

$x_1 = 2/3 \cdot 10 = \frac{20}{3} \approx 6.666$

find position $\bar{x}$ so that moments of masses:



formula to determine $\bar{x}$:

$$m_1(x_1-\bar{x}) + m_2(x_2-\bar{x}) + m_3(x_3-\bar{x})$$

$-5 \quad -4 \quad -3 \quad m_1 \quad m_2 \quad m_3 \quad m_4$

formula to detemine $\bar{x}$:

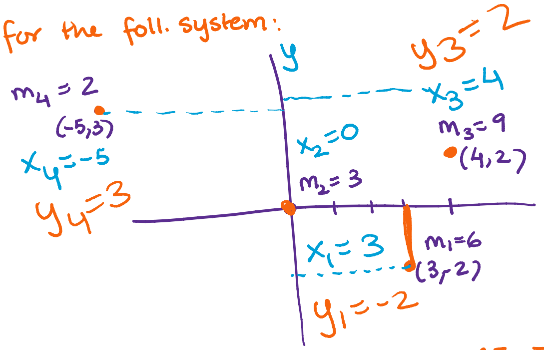
$$m_1(x_1-\bar{x}) + m_2(x_2-\bar{x}) + m_3(x_3-\bar{x}) + m_4(x_4-\bar{x}) = 0$$

$$\underbrace{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}_{\sum_{i=1}^{4} m_i x_i} - \underbrace{(m_1+m_2+m_3+m_4)}_{\sum_{i=1}^{4} m_i}\bar{x} = 0$$

$$\bar{x} = \frac{\sum_{i=1}^{4} m_i x_i}{\sum_{i=1}^{4} m_i} = \frac{10*(-5) + 15*0 + 5*4 + 10*7}{10+15+5+10}$$

$$CoM = \bar{x} = \frac{-50+0+20+70}{40} = 1$$

for the foll. system:



we want to calculate the CoM = $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{M_y}{M}, \quad \bar{y} = \frac{M_x}{M}$$

M_y = moments about y-axis $(x=0)$

M_x = " " x-axis

$M = m_1 + m_2 + m_3 + m_4 = 6+3+9+2 = 20$

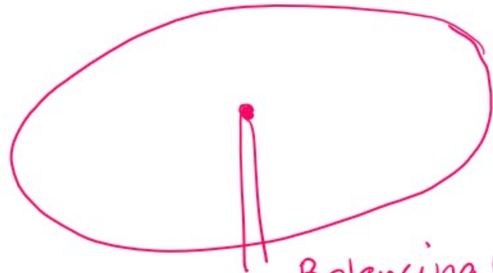
$$M_y = m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4$$
$$= 6*3 + 0 + 9*4 + 2*(-5)$$
$$= \underbrace{18+36}_{54} - 10 = 44$$

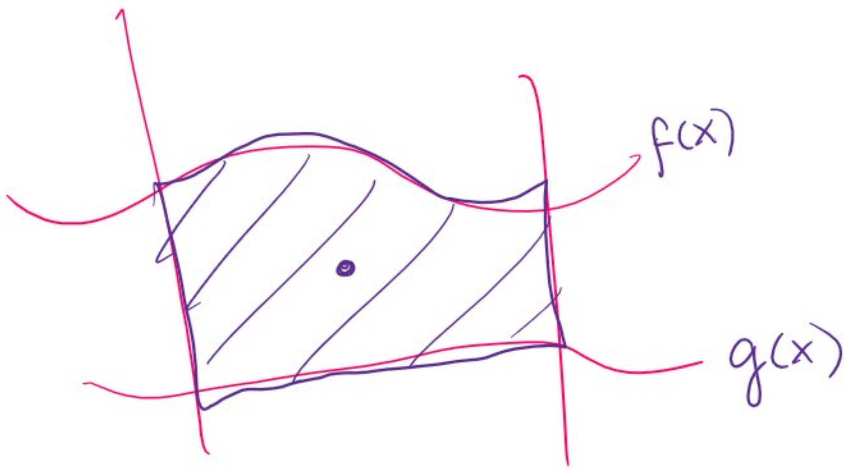
$$M_x = m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4$$
$$= 6*-2 + 0 + 9*2 + 2*3$$
$$= -12 + 18 + 6 = 12$$

$$CoM = \left(\frac{44}{20}, \frac{12}{20}\right) = \left(\frac{11}{5}, \frac{3}{5}\right) \text{ 1st Quad}$$

CoM:

CoM:





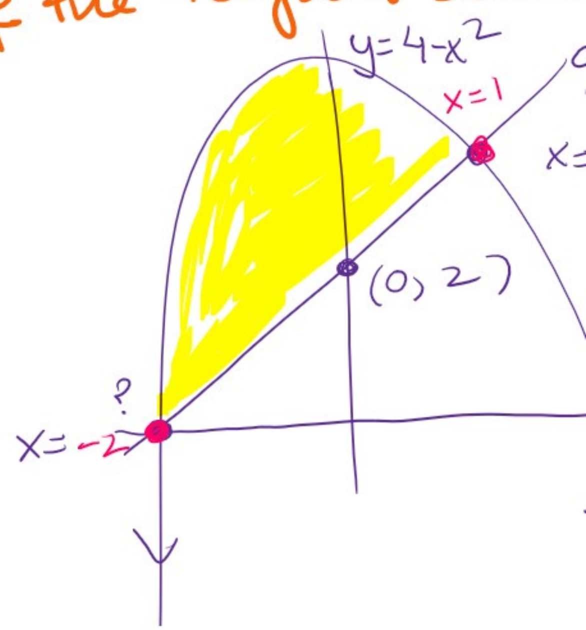
Discrete system $\{ (m_i, \vec{x}_i) : i = 1, 2, 3, 4 \}$

$\downarrow$

Continuous system

CoM for continuous sys = Centroid of region

example: find the centroid of the region bound[ed] by: $f(x) = 4 - x^2$, $g(x) = x + 2$.



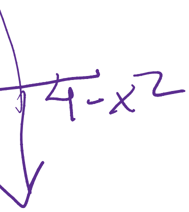
$$\text{area} = \int_a^b (4 - x^2) - (x+2)\, dx$$

$a = ?$ $\quad b = ?$

ed

$g(x) = x + 2$

$= 6$



$$4 - x^2 = x + 2$$

$$+x^2 \qquad +x^2$$

$$-4 \qquad -4$$

$$x^2 + x - 2 = 0 \Rightarrow (x-1)(x+2) = 0$$

$$x = -2, \quad x = 1.$$

mass = density * area