Math 1312

Thursday, October 24, 2019 3:02 PM

Work Done $W = FD$

Example: A force of 112 Newtons is required to move a cement block 8 m. How much work is done?

$W = 112 * 8$

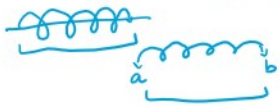
$= 896$ Newton meters.

Force is variable.

$F(x)$ $x=a$ $x=b$

$W = \int_a^b F(x)\,dx$

$F(x) = ?$



Work Done problems: how to determine $F(x)$?

for a spring, $F(x) = kx$ (Hooke's Law)

k: constant value no matter what displacement occurs, this value $\frac{F(x)}{x}$.

$$W = \int_a^b F(x)\,dx = \int_a^b kx\,dx = k\int_a^b x\,dx = k\frac{x^2}{2}\Big|_a^b$$

① A force of 20 lbs stretches a spring 9 inches. Find the work done in stretching the spring 12 inches from its natural position.

$W = \int_a^b F(x)\,dx$ ← fact / information

$W = \int_0^{12} F(x)\,dx$ (kx)

Use Force of 20 lbs stretches spring 9 inches.

F x

$F(9) = 20$

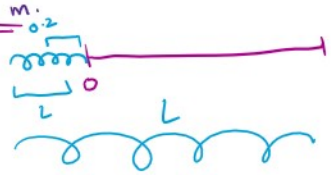
$9k = 20 \Rightarrow k = 20/9$

$$W = \int_0^{12} \frac{20}{9}x\,dx = \frac{20}{9}\frac{x^2}{2}\Big|_{x=0}^{12}$$

$$= \frac{10}{9} * 144$$

$W = 160$ pounds/inch

7.5 Newton-meter of work is required to compress a spring 0.2 meters from its natural length. Find the work required to compress the spring an additional 0.1 m.



$F(x) = kx$

Work Done $= \int_0^{0.2}$

Work Done $= \int_0^{0.2}$

$7.5 = \int_0^{0.2} F(x)\,dx$

$7.5 = \int_0^{0.2} kx\,dx$ Find k?

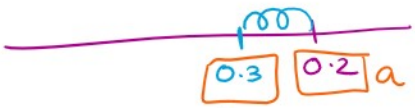
$7.5 = \int_0^{0.2} kx\,dx$ Find k?

$7.5 = k \int_0^{0.2} x\,dx = k \left.\frac{x^2}{2}\right|_0^{0.2}$

$7.5 = \frac{1}{2}(0.04)k \Rightarrow \frac{7.5 * 2}{0.04} = k$

$375 = k$

find work required in compressing it an additional 0.1 m.



$W = \int_a^b kx\,dx \qquad k = 375$

$= 375 \int_{0.2}^{0.3} x\,dx = 375 \left.\frac{x^2}{2}\right|_{x=0.2}^{0.3}$

$= \frac{375}{2}(0.09 - 0.04) = 9.375$ Nm.

Hint to homework: A force of 30 lbs stretches a spring 9 inches. Find the work done in stretching the spring 2 feet from its natural position.

#11

$F(9) = 30 \Rightarrow 9k = 30 \Rightarrow k = \frac{30}{9}$

$k = \frac{10}{3}$

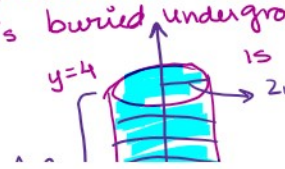
$W = \int_0^{24} \frac{10}{3} x\,dx$

we know that 1 foot = 12 inches.

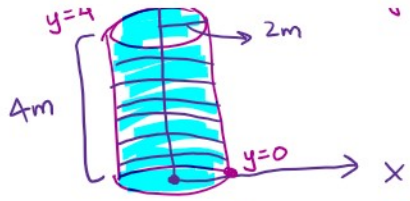
$= \frac{10}{3} \left.\frac{x^2}{2}\right|_0^{24}$

$F(x) = kx$ Hooke's Law

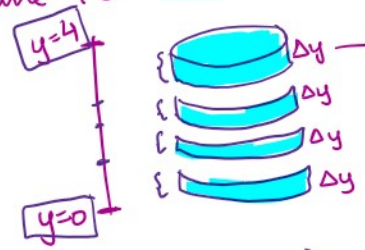
Problem: A cylindrical tank 4 meters high with radius 2m is buried underground so that the top of cylinder is at ground level.



cylinder



How much work is done in pumping a full tank of water to the ground level.
Assume that water has a weight of 9800 N/m3.



ΔF = force
= weight * volume of

$\Delta F = 9800 * \pi * 2^2 * \Delta y$

$\Delta F = 39,200\pi\ \Delta y$

ΔW = W. Done by 

= ΔF * Displacement

$= 39,200\pi\ (4-y)\ \Delta y$

16-8

16 - 4*2

39,200 * 8

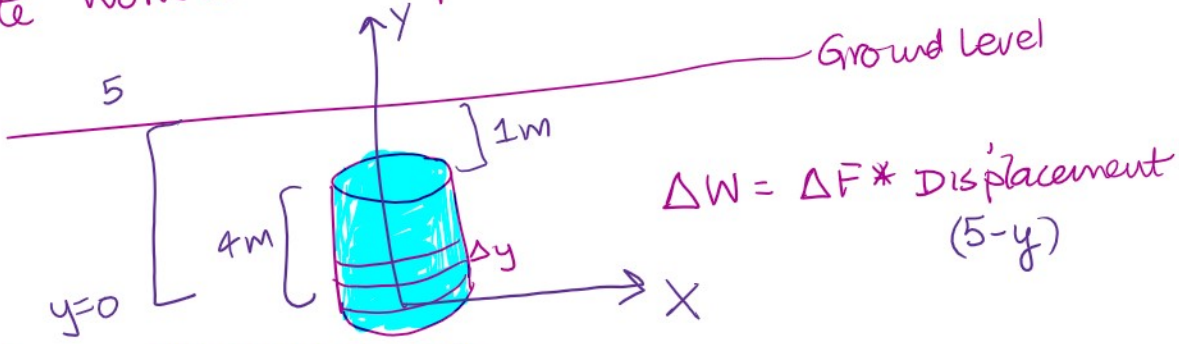
$$W = \int_{y=0}^{y=4} 39,200\pi\ (4-y)\ dy = 39,200\pi \left(4y - y^2/2\right)\Big|_0^4$$

$= 39,200\pi(16 - 4*2)$

$= 31,3600\pi$ N m

Suppose the cylindrical tank is buried underground so that the top of cylinder is 1 m below ground level.

Calculate Work done in pum... 9800 N m3



ΔW = ΔF * Displacement
(5-y)

ΔF = 9800 * Volume

$= 9800 * \pi * 2^2 * \Delta y$

$= 39,200\pi\ \Delta y$

$\int_{29}^{5} 200\pi * (5-y)\, dy$

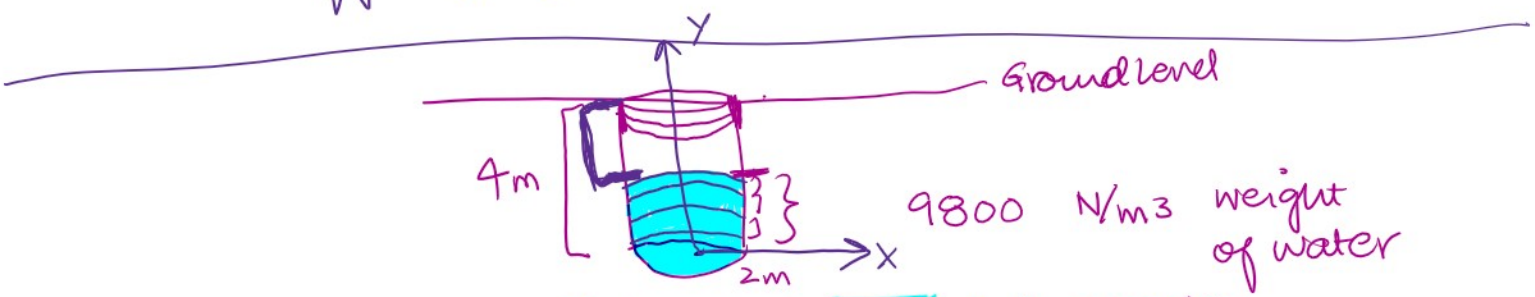
$$W = \int_0^5 39,200\pi * (5-y)\, dy$$

$$= 39,200\pi \left(5y - y^2/2\right)\Big|_{y=0}^{5}$$

$$= 39,200\pi \left(25 - \frac{25}{2}\right)$$

$$= 39,200\pi * \frac{25}{2}$$

$$W \approx 1,539,380.4 \text{ N m}$$



9800 N/m3 weight of water

Assume the cylindrical tank is half full of water. Buried underground so that the top is at ground level. Find the work done in pumping water to top.

$$W = \int_{y=0}^{2} \Delta w\, dy$$

$$\Delta w = \int_0^2 \Delta F * \text{Displacement}$$

(4-y)

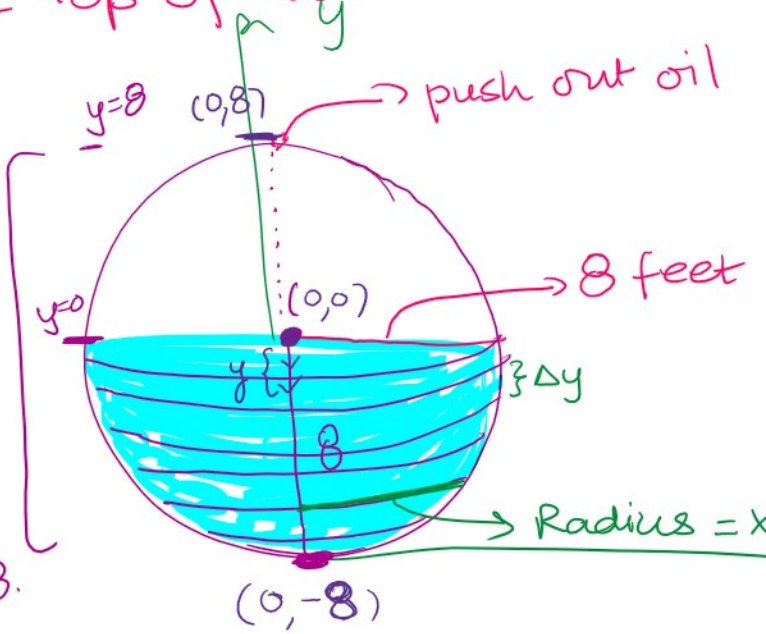
$$W = \int_0^2 39,200\pi\,(4-y)\, dy$$

complete it!

Example 4: A spherical tank of radius 8 feet is half full of oil that weighs 50 pounds per cubic foot. Find the work done in pumping all the oil out

full of oil

Find the work done in pumping all the oil out through a hole in the top of the tank.



Solution:

$\Delta F = 50 * \text{volume}$

$\Delta F = 50 * \pi * x^2 \Delta y$ → should be in terms of y!

$\Delta W = \Delta F * (16 - y)$, $0 \le y \le 8$.

$$W = \int_0^8 50\pi x^2 (16-y)\, dy$$

Can x^2 be expressed in terms of y?

$(x-0)^2 + (y-8)^2 = 16$

Complete 7.5, Review Problems