Arc Length

Tuesday, October 22, 2019 2:59 PM

$f(x) = \ln \sin x \qquad \frac{\pi}{4} \le x \le \pi/2$

arc length of $f(x)$ between $\pi/4$ & $\pi/2$

$$S = \int_{\pi/4}^{\pi/2} \sqrt{1 + (f'(x))^2}\, dx$$

$f(x) = \ln \sin x$

$f'(x) = \frac{1}{\sin x} * \cos x = \cot x$

$$S = \int_{\pi/4}^{\pi/2} \sqrt{1 + (\cot x)^2}\, dx$$

use Pythagorean Identity:

$1 + \cot^2 x = \operatorname{cosec}^2 x$

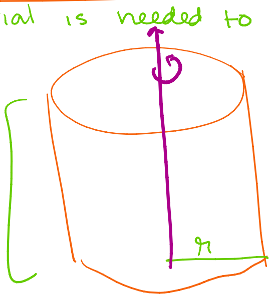
$$S = \int_{\pi/4}^{\pi/2} \sqrt{\operatorname{cosec}^2 x}\, dx$$

$$= \int_{\pi/4}^{\pi/2} \operatorname{cosec} x\, dx$$

$$S = -\ln|\operatorname{cosec} x + \cot x) \Big| \Big|_{x=\pi/4}^{\pi/2}$$

How much material is needed to cover the surface of the cylinder?

h



Lateral area:

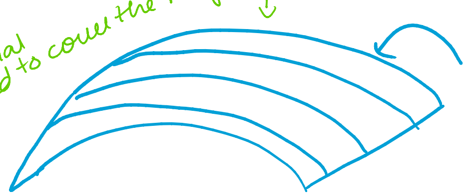
$2\pi r h$

$r \rightarrow$ distance from axis of revolution

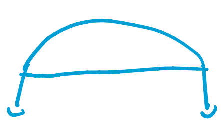
$h \rightarrow$ height of surface of solid / length of solid.

material needed to cover the roof?

#34



Roof



example from slides:

$$S = 2\pi \int_0^1 x^3 \sqrt{1 + (3x^2)^2}\, dx$$

$S = 2\pi \int_0^x \sqrt{\ldots}$

$$S = 2\pi \int_{x=0}^{1} x^3 \sqrt{1+9x^4}\, dx$$

u-substitution Let $u = 1+9x^4$

$\frac{du}{36} = \frac{36x^3\,dx}{36}$

$\frac{du}{36} = x^3 dx$

When $x=0$ $u = 1+0 = 1$

$x=1$ $u = 1+9 = 10$

$$S = 2\pi \int_{u=1}^{10} \sqrt{u}\, \frac{du}{36} = \frac{2\pi}{36} \int_1^{10} u^{1/2} du$$

$$= \frac{\pi}{18} \left. \frac{u^{3/2}}{3/2} \right|_{u=1}^{10}$$

using $\int u^n\, du = \frac{u^{n+1}}{n+1}$

$= \frac{\cancel{2}}{3} * \frac{\pi}{\cancel{18}_9} \left[10^{3/2} - 1 \right] = \frac{\pi}{27} \left(10^{3/2} - 1 \right)$

$= 3.5631$

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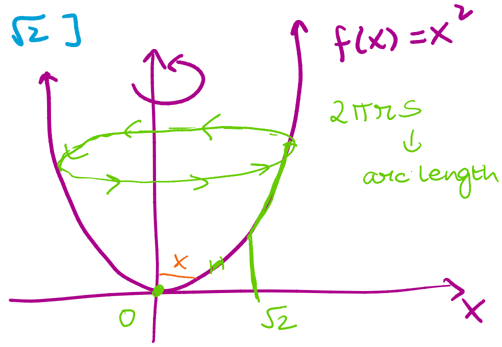
example: find area of surface formed by revolving the graph of $f(x) = x^2$ on the interval $[0, \sqrt{2}]$ about y-axis.



$$S = 2\pi \int_0^{\sqrt{2}} r(x)\, s(x)\, dx$$

$r(x)$: distance from y-axis

$r(x) = x$

$s(x)$: arc length

$$S = 2\pi \int_0^{\sqrt{2}} x \sqrt{1+(f'(x))^2}\, dx$$

$f(x) = x^2$

$f'(x) = 2x \rightarrow f'(x)^2 = 4x^2$

$$= 2\pi \int_{x=0}^{\sqrt{2}} x \sqrt{\underbrace{1+4x^2}}\, dx$$

$x\,dx =$

$$= 2\pi \int_{x=0} x \sqrt{1+4x^2}\, dx$$

$$\boxed{1+4x^2 = u}$$

$$4(2x)dx = du$$

$$\frac{8x\,dx}{8} = \frac{du}{8}$$

$$x\,dx = \frac{du}{8}$$

$\boxed{x=0} \Rightarrow u = 1+0 = 1$

$\boxed{x=\sqrt{2}} \Rightarrow u = 1 + 4*(\sqrt{2})^2 = 1 + (4*2) = 9$

$$S = 2\pi \int_{u=1}^{9} \sqrt{u}\, \frac{du}{8}$$

$$= \frac{\pi}{4} \int_1^9 u^{1/2}\, du$$

$$= \pi/4 \left. \frac{u^{3/2}}{3/2} \right|_{u=1}^{9}$$

$$= \frac{2}{3}\pi/4 \left(9^{3/2} - 1\right)$$

$$= \pi/6 \,(27-1)$$

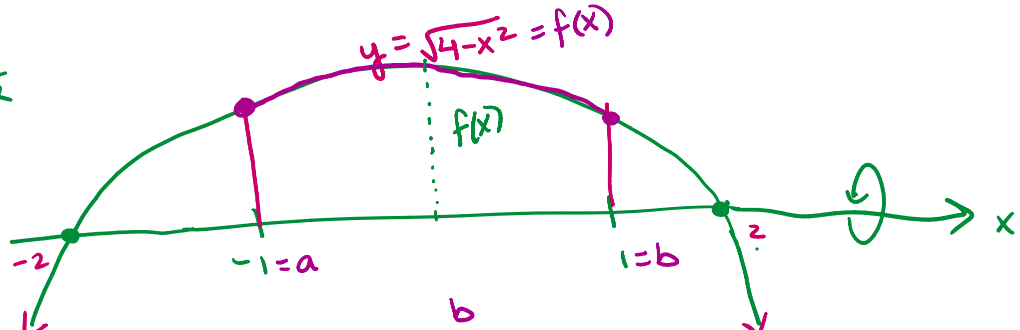
$$= 13\pi/3 = 13.614$$

$$9^{3/2} = (3^2)^{3/2} = 3^3 = 27$$

#43 $\boxed{y = \sqrt{4-x^2}}$ $\quad -1 \le x \le 1$

for the given curve y, write and evaluate the definite integral that represents the area of the surface generated by revolving y about $\boxed{\text{x-axis}}$ on the indicated interval.

$\sqrt{x^2}$



$$S = 2\pi \int_a^b r(x)\, s(x)\, dx$$

↳ arc length of f(x) bet. -1 and 1.

$\sqrt{1+f'(x)^2}$

a

distance from x-axis

bet. −1 and 1.

$\sqrt{1+f'(x)^2}$

$$S = 2\pi \int_{-1}^{1} \sqrt{4-x^2}\,\sqrt{1+f'(x)^2}\;dx$$

$$f(x) = \sqrt{4-x^2} = (4-x^2)^{1/2}$$

$$f'(x) = \frac{1}{2}(4-x^2)^{1/2-1}\,\underbrace{\frac{d}{dx}(4-x^2)}$$

CHAIN RULE

$\frac{d}{dx}x^n = nx^{n-1}$

$$= \frac{1}{\cancel{2}}(4-x^2)^{-1/2}\,(0-\cancel{2}x)$$

$$= (4-x^2)^{-1/2}(-x)$$

$$f'(x) = \frac{-x}{(4-x^2)^{1/2}}$$

$$f'(x)^2 = \frac{x^2}{4-x^2}$$

$$S = 2\pi \int_{-1}^{1} \sqrt{4-x^2}\,\sqrt{1+\frac{x^2}{4-x^2}}\;dx$$

$$= 2\pi \int_{-1}^{1} \sqrt{4-x^2}\,\sqrt{\frac{4-x^2}{4-x^2}+\frac{x^2}{4-x^2}}\;dx$$

$$= 2\pi \int_{-1}^{1} \sqrt{4-x^2}\,\sqrt{\frac{4-x^2+x^2}{4-x^2}}\;dx$$

$$= \int_{-1}^{1} \cancel{\sqrt{4-x^2}}\,\sqrt{4}\;dx$$

$$= 2\pi \int_{-1}^{1} \cancel{\sqrt{4-x^2}} \frac{\sqrt{4}}{\cancel{\sqrt{4-x^2}}} dx$$

$$= 2\pi \int_{-1}^{1} 2 \, dx = 4\pi \int_{-1}^{1} dx.$$

$$= 8\pi.$$