CPS 5310

Tuesday, April 28, 2020 1:38 PM

Goal : analyze LV equation , LV-solver numerical

$$\frac{dx}{dt} = ax - cxy = x(a-cy) \longrightarrow a/c \qquad a,b,c,d > 0$$

$$\frac{dy}{dt} = -by + dxy = (-b+dx)\,y$$

Equilibrium points / Fixed Points $(0,0)$ $\left(\frac{b}{d}, \frac{a}{c}\right)$

$$JF(x,y) = \begin{bmatrix} a-cy & -cx \\ dy & -b+dx \end{bmatrix}$$

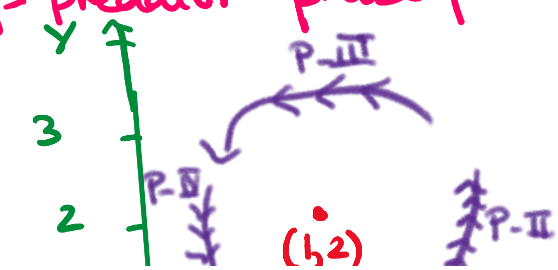
$$JF(0,0) = \begin{bmatrix} a & 0 \\ 0 & -b \end{bmatrix} \qquad \left.\begin{aligned}&(a-\lambda)(-b-\lambda) = 0 \\ &\Rightarrow \lambda = a, \ \lambda = -b\end{aligned}\right\} \text{UNSTABLE}$$

$$JF\left(\frac{b}{d}, \frac{a}{c}\right) = \begin{bmatrix} 0 & -bc/d \\ ad/c & 0 \end{bmatrix} \qquad \left.\begin{aligned}\lambda^2 + ab &= 0 \\ \lambda &= \pm i\sqrt{ab}\end{aligned}\right\} \text{STABLE}$$

specific choices of a, b, c, d to determine the phase portrait. $a=4$ $b=3$ $c=2$ $d=3$

$(b/d, a/c) = (1,2)$

4 phases for evolution of prey-predator phase portraits



Phase I:

y pop^n is low $\Rightarrow$ x pop^n $\uparrow$

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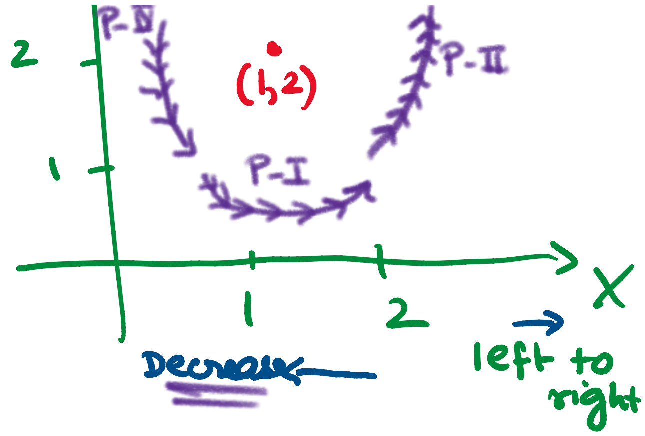


Phase II :
Prey pop^n HIGH $\Rightarrow$ y pop^n $\uparrow$

Phase III
Pred pop^n HIGH $\Rightarrow$ Prey pop^n $\downarrow$

Phase IV
Low prey pop^n $\Rightarrow$ Pred pop^n $\downarrow$

Look at lv_solver.m & perturbations of data.

- impact of $\uparrow$ prey production rate ($\uparrow a$)
- impact of $\uparrow$ predator death rate ($\downarrow b$)

$\to$ $a+b+c+d = S$ $\quad a = 0.7 \quad b = 0.2 \quad c = 0.05 \quad d = 0.05$

$\to$ $b = 0.7 \quad a = 0.2 \quad c = d = 0.05$

Oil production versus oil prices (App^n)

Drawbacks of LV-model

1. Too simple with x, y more species should be considered. (Plant $\to$ herbivore $\to$ carnivore)

2. $$\begin{cases} \dfrac{dx}{dt} = ax - cxy - \dfrac{a}{K_1}x^2 \\[2ex] \dfrac{dy}{dt} = -by + dxy + \dfrac{b}{K_2}y^2 \end{cases}$$

$c = 0 \to x(t) = x_0 e^{at}$ unrealistic

include logistic growth term.

K_1, K_2 are carrying capacities of x & y respectively

$$ry(1 - y/K)$$

Include more species : $x_1, x_2, \ldots x_n$ n-species

Include more species : $x_1, x_2, \dots x_n$ n species

$i = 1, 2, \dots n$

$$\frac{dx_i}{dt} = x_i \left(\sum_{j=1}^{n} A_{ij} (1 - x_j) \right)$$

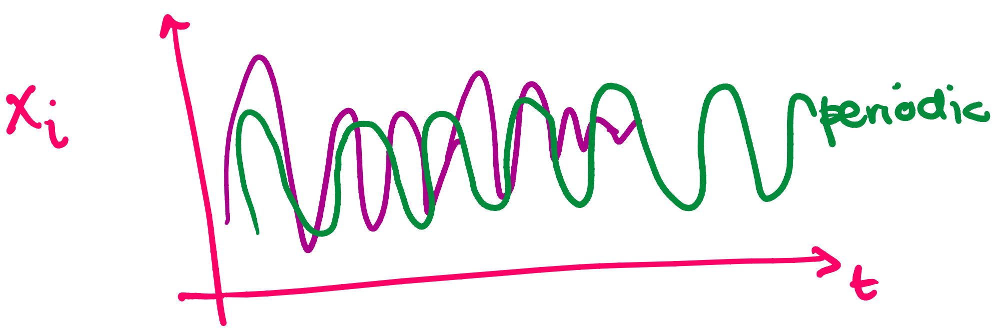
where

A is $n \times n$ matrix.

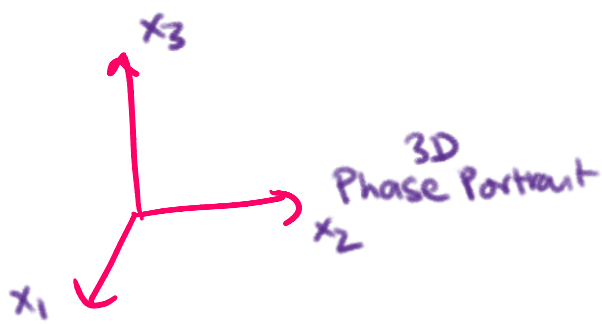
example: $n = 3$

$$A = \begin{bmatrix} 0.5 & 0.5 & 0.1 \\ -0.5 & -0.1 & 0.1 \\ \gamma & 0.1 & 0.1 \end{bmatrix}$$

$\gamma \to$ parameter that can be tuned to get a range of popⁿ behavior.



$n = 3$



Parameter Estimation : H $\to$ Hare L $\to$ Lynx

Lv_data.m

$$\begin{cases} \dfrac{dx}{dt} = ax - cxy \\[2ex] \dfrac{dy}{dt} = -by + dxy \end{cases}$$

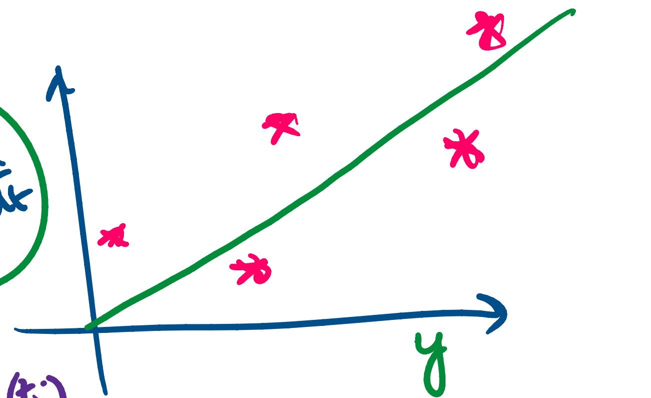
ode43

@logistic-growth

$$\frac{dx}{dt} = (a-cy)\,x \rightarrow$$

know $x(t_i), y(t_i)$

$a - cY$



polyfit

$$\frac{1}{x}\frac{dx}{dt} = \underbrace{a-cY}$$

"Y"

$$\frac{1}{x}\frac{dx}{dt}$$

$$\frac{dx_i}{dt} \approx D_h x(t_i)$$

$$\frac{1}{y}\frac{dy}{dt} = -b + dx$$

exam # 2(c)

$$dx/dt = \underbrace{rK - rx} \approx \overset{\curvearrowright rK}{b} + \underset{\downarrow\,-r}{m}x$$

p = polyfit$(x,y,1)$

$p(2)x + p(1)$ wrong

$p(1)x^1 + p(2)$