Hwk03key_Hwk4hint

Tuesday, April 21, 2020 3:13 PM

Tex mistakes:

\begin{center}

\end{center} is the wrong environment for math eqns or formula.

&= is an alignment tab within align environment

\begin{align*}
\end{align*}

$ math formula $

$ \lambda $

underscore → _

\begin{align*}
\begin{bmatrix}
\end{bmatrix}
\end{align*}

$$\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}$$

$ X_12 $ → $ X_12 $

$ X_{12} $ → $ X_{12} $

Homework 04/2(c):

$$\theta'' + \theta' + 4\sin\theta = 0$$

$$x_1 = \theta$$

(0,0) eqbm point

$$x_1' = x_2$$

$$x_2' = -4\sin x_1 - x_2$$

Eqbm points:

$x_2 = 0$ { → $x_2 = 0$

Eqbm points:

$$\left.\begin{aligned} x_2 &= 0 \\ -4\sin x_1 - x_2 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} x_2 &= 0 \\ -4\sin x_1 &= 0 \rightarrow \sin x_1 = 0 \end{aligned}$$

$$\sin x_1 = 0 \Rightarrow x_1 = 0$$

$$\Downarrow$$

$$x_1 = n\pi \qquad n \text{ is an integer.}$$

Family of equilibrium points: $\left\{\begin{pmatrix} n\pi \\ 0 \end{pmatrix} : n \text{ is an integer}\right\}$

$$JF(x_1, x_2) = \begin{bmatrix} 0 & 1 \\ -4\cos x_1 & -1 \end{bmatrix}$$

$$JF(n\pi, 0) = \begin{bmatrix} 0 & 1 \\ -4(-1)^n & -1 \end{bmatrix}$$

$\sin n\pi = 0$ n being integer

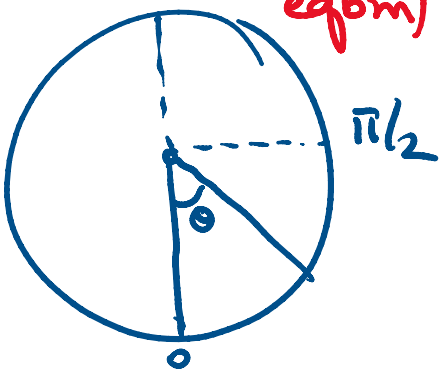
$$= \begin{bmatrix} 0 & 1 \\ 4(-1)^{n+1} & -1 \end{bmatrix}$$

2 set of eigen values:

e.value corr. to (0,0):

$$\lambda_{\pm} = \underbrace{-0.5 \pm i\, 1.9365}$$

complex no. & its conjugate



eigenvalues are complex numbers $-0.5 \pm i\, 1.9365$.

If real part of complex evalue is negative

If real part of complex evalue is negative

then, we have a stable eqbm pt.

(Critical-pts.pdf)

$n \neq 0$ $\quad JF(0,0)$

$$JF(n\pi,0) = \begin{bmatrix} 0 & 1 \\ -4\cos n\pi & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4(-1)^n & -1 \end{bmatrix}$$

$$JF(n\pi,0) = \begin{bmatrix} 0 & 1 \\ 4(-1)^{n+1} & -1 \end{bmatrix}$$

$n+1$ is odd

$n=0$

$$\begin{bmatrix} 0 & 1 \\ -4 & -1 \end{bmatrix}$$

$\downarrow$

$-0.5 \pm i\, 1.9365$

$n \neq 0$, $n+1$ is even

$$\begin{bmatrix} 0 & 1 \\ 4 & -1 \end{bmatrix}$$

$\downarrow$

2 eigen values

real but having opposite signs