Cps 5310

Tuesday, April 14, 2020 1:53 PM

$$\frac{dy}{dt} = ry(1 - y/k) \qquad y(0) = y_0$$

Does not require exact solution $y(t)$ information.

t_i	y_i

Idea: $$\frac{1}{y}\frac{dy}{dt} = \underbrace{r - \frac{r}{k}y}_{Y = mx + b}$$

where $b = r$
$m = -r/k$

Based on (t_i, y_i), we can determine $Y = mx + b$

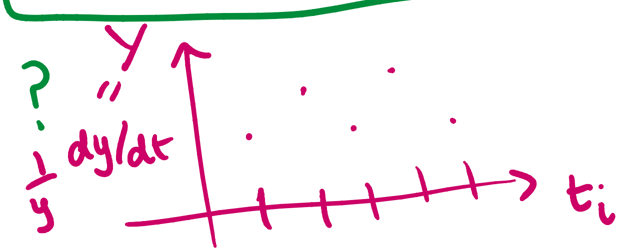
$r = b$

And using $b = r$, $m = -r/k$ $\Rightarrow$ $\boxed{k = -b/m}$

$Y = mx + b$ using data $(x = t_i, y_i)$

$$Y = mx + b \rightarrow \boxed{Y_i = 1/y_i \frac{dy}{dt}(t_i)}$$

We dont know $\frac{dy}{dt}(t_i) = ?$



So instead, $\frac{dy}{dt}(t_i) \approx D_i^+ y(t_i) = y(t_i + h) - y(t_i)$

So instead, $\frac{dy(t_i)}{dt} \approx D_h^+ y(t_i) = \frac{y(t_i+h) - y(t_i)}{h}$

$$\text{polyfit}\left(t_i, \underbrace{\frac{1}{y_i} D_h^+ y(t_i)}, 1\right)$$

$$\longrightarrow \frac{1}{y}\frac{dy}{dt} = r - \frac{r}{k} y$$

(See forward_difference_pe.m)

Central Difference to approximate $\frac{dy}{dt}$

$$\frac{dy}{dt} \approx D_h y(t) = \frac{y(t+h) - y(t-h)}{2h}$$

$$\frac{dy}{dt} = D_h y(t) + O(h^2)$$

In contrast to $\frac{dy}{dt} = D_h^+ y(t) + O(h)$

Index	decades t_i	y_i pops
1	0	y_0
2	10	$y(t_1)$
3	20	y_2

$h=10$ $h=10$

$$D_h y(t_1) = \frac{y(t_1+h) - y(t_1-h)}{2h}$$

$$= \frac{y_2 - y_0}{2*10}$$

$$D_h y(t_1) = \frac{y_2 - y_0}{20}$$

System of ODEs:

System of ODEs

$$\begin{cases} \dfrac{dy_1}{dt} = f_1(t, y_1, y_2) \\[2ex] \dfrac{dy_2}{dt} = f_2(t, y_1, y_2) \\[2ex] y_1(0) = y_{01}, \quad y_2(0) = y_{02} \end{cases}$$

$$Y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \qquad \frac{dY}{dt}(t) = \begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} \qquad y_i'(t) = \frac{dy_i}{dt}$$

Compact form representation of sys. of ODES:

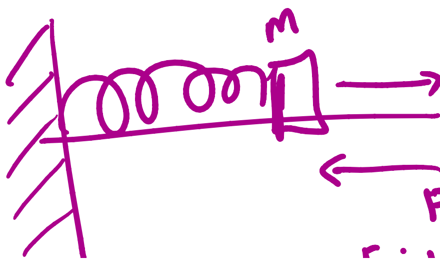
$$Y'(t) = \frac{dY}{dt} = F(t, Y)$$

here $\quad F(t, Y) = \begin{bmatrix} f_1(t, y_1, y_2) \\ f_2(t, y_1, y_2) \end{bmatrix}$

$$\left.\begin{array}{l} dy_1/dt = f_1 \\ dy_2/dt = f_2 \\ y_1(0) = y_{01}, \quad y_2(0) = y_{02} \end{array}\right\} \Rightarrow \begin{array}{l} Y' = F(t, Y) \\ Y(0) = Y_0, \quad Y_0 = \begin{bmatrix} y_{01} \\ y_{02} \end{bmatrix} \end{array}$$

Harmonic Oscillator:

(look at lecture notes on harmonic oscillator)



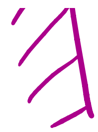
m

horizontal

$\longrightarrow x$ (displacement $= x$ units)

$\longleftarrow$

$F = -kx, \quad k > 0$



$F = -kx \ , \ k > 0$

Friction $= -b\dfrac{dx}{dt} \ , \ b > 0$

Total force = mass * acceleratn

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

Second order ODE

second order derivative $\Rightarrow$

$$y_1 = x \ , \quad y_2 = \frac{dx}{dt} \Rightarrow y_2' = \frac{d^2x}{dt}$$

$$\frac{dy_1}{dt} = y_2$$

$$m y_2' + b y_2 + k y_1 = 0$$

$$\frac{dy_2}{dt} = -\frac{1}{m}\left(by_2 + ky_1\right)$$

$$Y = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \qquad F(t,Y) = \begin{bmatrix} y_2 \\ -\frac{1}{m}(by_2 + ky_1) \end{bmatrix}$$

Vector-Valued ODE

$$\frac{dY}{dt} = F(t,Y) \qquad Y \in \mathbb{R}^2$$

y_2

$$F(t,y) = \begin{bmatrix} y_2 \\ -\frac{b}{m}y_2 - \frac{k}{m}y_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= A y$$

Eigen values & vector of A & how it relates to Phase Lines (in two dimensn)