CPS-5310 Curve fitting

Thursday, April 9, 2020 2:23 PM

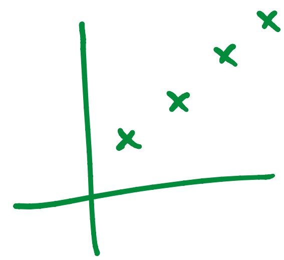
$x_i \quad y_i \quad i=1,\dots,N$

$$\min_{m,b} \; E(m,b) = \sum_{i=1}^{N} \left(y_i - P_k(x_i) \right)^2$$

$P_k(x) = mx + b$

Instead of poly $P_k(x)$ polyfit(x, y, 2)

$(x_i \; y_i)$ → US popⁿ census data.



population models (ODE)

$P_k(x)$ → use closed form solution

$$\frac{dY}{dt} = kY, \quad Y(0) = y_0$$

$$\min_{k, y_0} \; E(k, y_0) = \sum_{i=1}^{N} \left| y_i - \underbrace{y_0 e^{k x_i}}_{\text{Non linear functn}} \right|^2$$

y_0 ↳ known

Minimizing $E(k, y_0)$ to detemine growth rate k & y_0. Non linear minimizatn

lsqcurvefit

codes folder of repo

look at (lsqcurvefit-pe.m)

Least squares curve fit requires knowledge of exact solution.

Regression with $\boxed{\text{Numerical Derivatives}}$:

$$\frac{dy}{dt} = ry\left(1 - y/K\right), \quad y(0) = y_0.$$

$$\frac{dy}{dt} = ry\left(1 - y/K\right), \quad y(0) = y_0$$

t_i	y_i

Given (t_i, y_i) $i = 1, 2, \ldots N$

find r, K, y_0

Forward Difference Parameter Estimation

Replace $\frac{dy}{dt} \to D_h y(t_i)$ $h \to$ stepsize

$$= \frac{y(t_i + h) - y(t_i)}{h}$$

$$\frac{y(t_i + h) - y(t_i)}{h} = r\, y(t_i)\left(1 - \frac{y(t_i)}{K}\right)$$

r, K unknown

$$\underbrace{\frac{1}{y(t_i)}\left(\frac{y(t_i + h) - y(t_i)}{h}\right)}_{} = \underbrace{r - \frac{r}{K}\, y(t_i)}_{\text{polyfit to figure out}} \approx mx + b$$

polyfit to figure out

r & $-r/K$

b = intercept → r; $-r/K$ → slope m

see forward-difference-pe.m code.