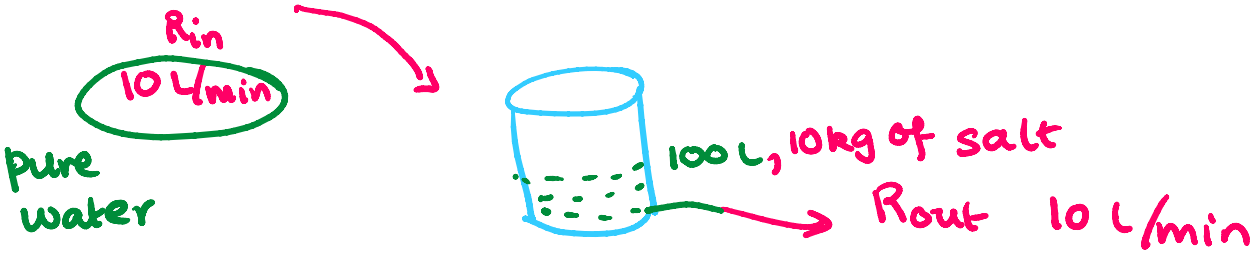
Cps-5310

Thursday, March 12, 2020 1:58 PM



at $t = 30$ min

what is the amt of salt?

Math model: (S, Q, M)

S → tank containing 100L of water, 10 kg of salt initially.

Let $S(t)$ denote amt of salt at time t.

Q → $S(30) = ?$

M →

$$\frac{ds}{dt} = \text{Rate of salt In} - \text{Rate of salt out}$$

$$\frac{ds}{dt} = 0 - \text{Rate of salt out}$$

$$\frac{ds}{dt} = -\frac{S(t)}{V(t)} * R_{out}$$

Concentration * Rout

$V(t)$ → volume of brine at time t

$$V(t) = 100 + (R_{in} - R_{out})t \qquad R_{in} = 10, R_{out} = 10$$

$$= 100$$

$S \equiv S(t)$

$$\frac{ds}{dt} = -\frac{S}{100} * 10 = -\frac{1}{10}S = F(t, s) = -\frac{S}{10}$$

ODE:

$$\frac{ds}{dt} = -\frac{S}{10}$$ (separable, autonomous)

$$\rightarrow \int \frac{ds}{s} = -\int \frac{dt}{10}$$

$$\rightarrow \int \frac{ds}{s} = -\int \frac{dt}{10}$$

$$\ln|s| = -\frac{t}{10} + c$$

$$\exp(\ln|s|) = \exp(-t/10 + c)$$

$$s(t) = e^{-t/10 + c}$$

$s(0) = 10$ kg $\quad 10 = e^{-0/10 + c}$

$$\Rightarrow \quad 10 = e^{c}$$

$$s(t) = e^{-t/10 + c} = 10e^{-t/10}$$

Check: $s(0) = 10e^{0} = 10$

$$s'(t) = 10 * \left(\frac{-1}{10}\right) e^{-t/10} = -e^{-t/10}$$

$$s(30) = 10e^{-30/10} = 10e^{-3} \text{ kg}$$

Q.2: $\quad \frac{ds}{dt} = 0.1 - \frac{s}{10}$

$F(t,s)$ → t doesn't appear explicitly

$\frac{ds}{dt} = 0.1 - \frac{s}{10}$ → 1/10 is Separable.

mult. by dt & divide by $0.1 - s/10$ (assume $0.1 - \frac{s}{10} \neq 0$)

mult. by dt & divide by

$$\int \frac{ds}{0.1 - s/10} = \int dt$$

$$\int \frac{ds}{(0.1 - s/10)} = t + c$$

Let $u = 0.1 - s/10$ $\quad du = -ds/10$

$-10\,du = ds$

$$\int \frac{-10\,du}{u} = t + c$$

$$-10 \ln|u| = t + c$$

$$\ln|u| = -\left(\frac{t+c}{10}\right)$$

$$0.1 - \frac{s}{10} = u = e^{-\left(\frac{t+c}{10}\right)}$$

$$\frac{s}{10} = 0.1 - e^{-\left(\frac{t+c}{10}\right)}$$

$$s(t) = 10\left(0.1 - e^{-\left(\frac{t+c}{10}\right)}\right) = 1 - 10e^{-\left(\frac{t+c}{10}\right)}$$

use $s(0) = 10$ to determine c.

$$10 = 1 - 10e^{-c/10}$$

$$10e^{-c/10} = 1 - 10 = -9$$

$$e^{-c/10} = -9/10$$

$10 e^{-c/10} = -9/10$

$$S(t) = 1 - 10e^{-t/10}(-9/10)$$

Another way:

$$\frac{ds}{dt} = 0.1 - \frac{s}{10} = \frac{1}{10} - \frac{s}{10} = \frac{1-s}{10}$$

$$\frac{ds}{1-s} = \frac{dt}{10} \Rightarrow -\ln|1-s| = t/10 + c$$

3rd Approach: $\frac{ds}{dt} = 0.1 - 0.1s$

$$\frac{ds}{dt} + ps = q$$

$p(t)$, $q(t)$ are functns of t.

using Integrating factor: $\mu \rightarrow \mu' = \mu p$

$$\mu \frac{ds}{dt} + \underbrace{\mu p}_{\mu'} s = \mu q$$

$$\frac{d}{dt}(\mu s) = \mu q$$

$$\mu s = \int \mu q \, dt + c$$

$$s(t) = \int \mu q \, dt + c$$

$$s(t) = \frac{\int \mu q \, dt + c}{\mu}$$

where μ satisfies $\mu' = \mu p$

$$\mu = e^{\int p \, dt}.$$

BitBucket NOT GITHUB

Homework Instructions (Code Submission)

lastname-cps5310 repository

↳ Hwk01

↙

{ lastname_p1.mac
p3.mac
p4.mac }

Population Models