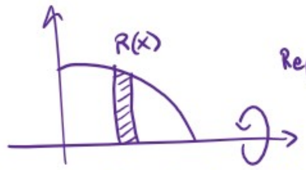
7.2 Homework 14, 18, 23, 32, 34

Thursday, October 10, 2019 3:08 PM

so far,



Representative Rectangle perp. to axis of revolution

Axis of Revolution is Vertical Axis

Find the volume of the solid formed by revolving the region bounded by the graphs of:

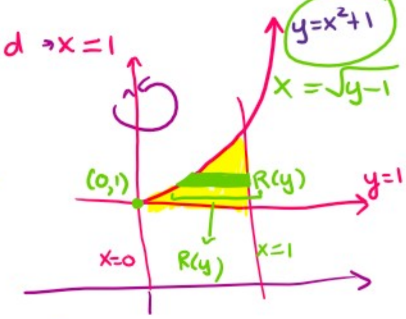
$\rightarrow y = x^2+1$

$\rightarrow y = 1$

$\rightarrow x = 0$ and $\rightarrow x = 1$

about y-axis.

Volume $= \pi \int_c^d R(y)^2\, dy$



we need y-values but we given x values.

Need: write $y = x^2+1$ as

$x = \boxed{} \rightarrow$ functn of y

$y = x^2+1$

$x^2+1 = y$

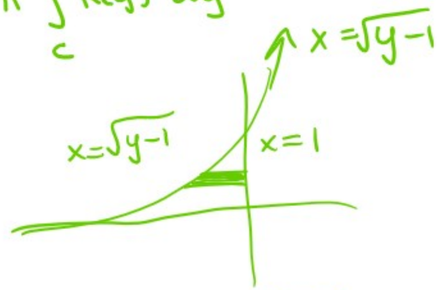
$-1 \quad -1$

$x^2 = y-1 \longrightarrow x = \sqrt{y-1}$

we have $x = \sqrt{y-1}$, $x = 0$ and $x = 1$

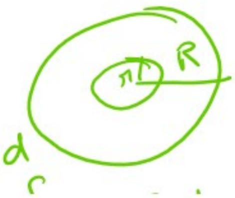
Volume of solid $= \pi \int_c^d (R(y)^2 - r(y)^2)\, dy$

instead of just $\pi \int_c^d R(y)^2\, dy$



$R(y) = 1$ and $r(y) = \sqrt{y-1}$

Washer method Difference of 2 volumes



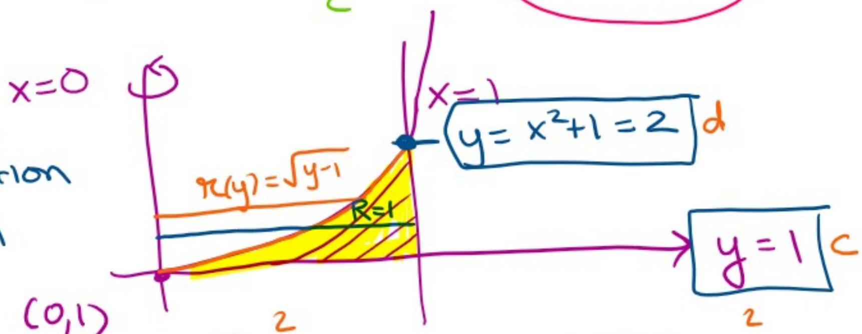
$\int^d R(y)^2\, dy$ $\qquad \int^d$

$$\text{Volume} = \pi \int_c^d R(y)^2\,dy - \pi \int_c^d r(y)^2\,dy$$

$$= \pi \int_c^d 1^2\,dy - \pi \int_c^d \left(\sqrt{y-1}\right)^2 dy$$ correction!

$r(y)$ = Distance from axis of Revolution

$R(y)$ = Distance from axis of Rev.



$$\text{Volume} = \pi \int_1^2 1^2\,dy - \pi \int_1^2 (y-1)\,dy$$

$$= \pi - \pi \frac{y^2}{2}\Big|_1^2 + \pi$$

$$= 2\pi - \pi\left(2 - \frac{1}{2}\right) = 2\pi - \frac{3\pi}{2} = \frac{\pi}{2}$$

one more: #13 7.2

Given: $y = \sqrt{x}$

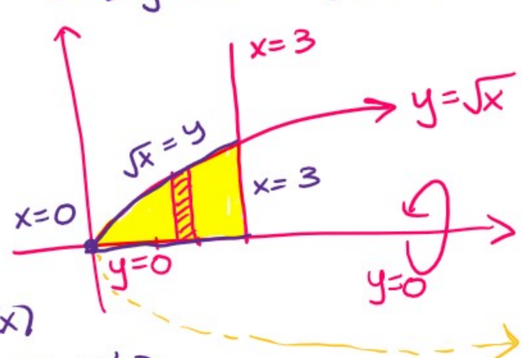
$y = 0$, $x = 3$

Find volume of solid formed by revolving graphs of above curves about

(a) x-axis (b) y-axis (c) $x = 3$ (d) $x=6$

Solution

(a)



$y = \sqrt{x}$ TOP $R(x)$

$y = 0$ BOTTOM $r(x)$

Points of intersection: $\sqrt{x} = 0 \Rightarrow x = 0$

$y=0$

Points of intersection : $\sqrt{x}=0 \Rightarrow x=0$

$a=0 \qquad b=3$

$$\text{Volume} = \pi \int_a^b \left(\underset{\sqrt{x}}{R(x)^2} - \underset{0}{r(x)^2}\right) dx$$

$$= \pi \int_0^3 (\sqrt{x})^2 - 0 \, dx = \pi \int_0^3 x\,dx$$

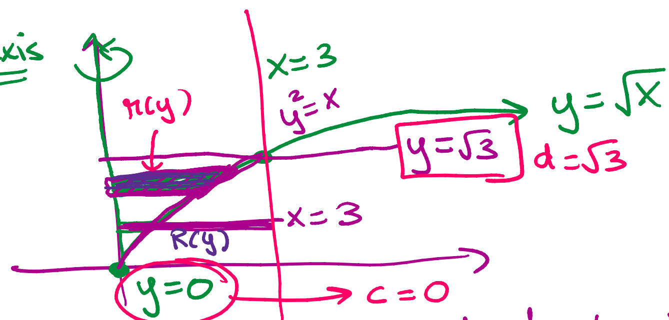
$$= \frac{9\pi}{2}$$

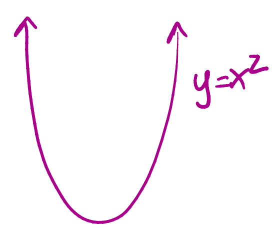
(b) about y-axis

$y=\sqrt{x} \qquad y=0 \qquad x=3$

Vol = $\Rightarrow \pi \int_c^d \square \, dy$

functn of y





$y=\sqrt{x} \longrightarrow$ needs to converted to x = functn of y

$y^2 = x$

$\boxed{x=y^2} \qquad \boxed{x=3}$ given

$$V = \pi \int_c^d R(y)^2 - \pi \int_c^d \underset{y^2}{r(y)^2}\,dy$$

$$= \pi \int_0^{\sqrt{3}} 3^2 - \pi \int_0^{\sqrt{3}} (y^2)^2 \, dy$$

$$= \pi \int_0^{\sqrt{3}} (9 - y^4)\, dy$$

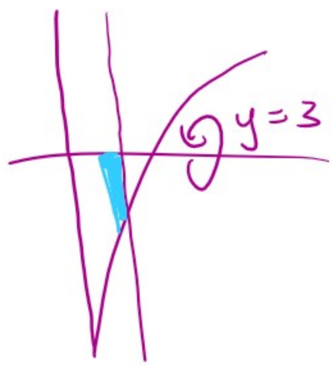
$$V = \pi\left(9\sqrt{3} - \frac{(\sqrt{3})^5}{5}\right) = \frac{36\sqrt{3}}{5}\pi$$

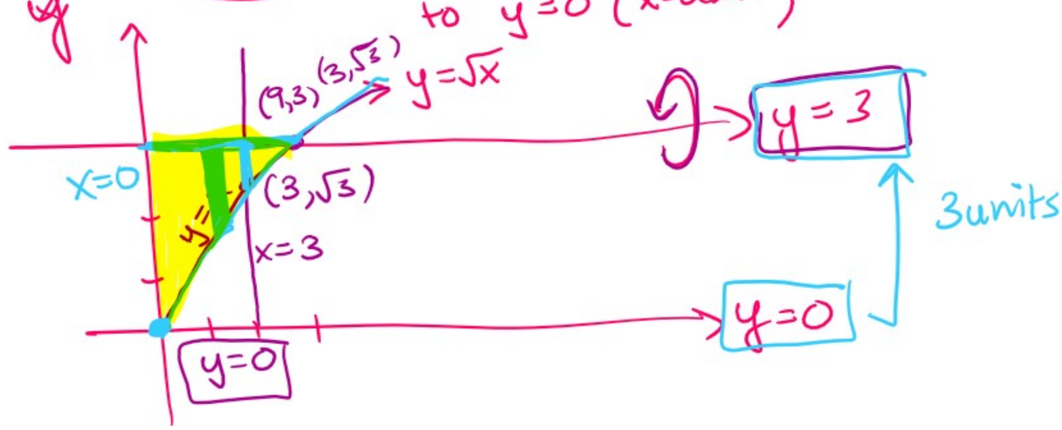
$$V = \pi\left(9\sqrt{3} - \frac{(\sqrt{3})^5}{5}\right) = \frac{36\sqrt{3}}{5}\pi$$

(c) $y=\sqrt{x}$ $y=0$ $x=3$

about the line $y=3$

horizontal axis parallel to $y=0$ (x-axis)





$R(x) = 3-0$ $r(x) = 3-\sqrt{x}$

$a=0$ $b=3$

$$\text{Volume} = \pi\int_0^3 \left(3^2 - (3-\sqrt{x})^2\right)dx$$

$$= 9\pi * 3 - \pi\int_0^3 (9 - 6\sqrt{x} + x)\,dx$$

$$= 27\pi - \pi(27) + 6\pi\frac{3^{3/2}}{3/2} - \pi\frac{9}{2}$$

$$= \frac{6*2}{\cancel{3}}\pi\,\cancel{3}\sqrt{3} - \frac{9\pi}{2} = 51.1596$$

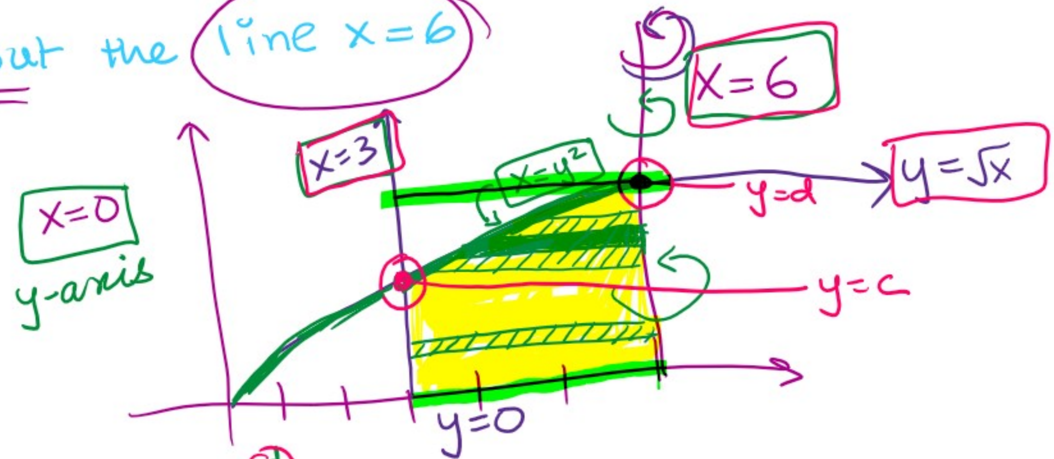
(d) $y=\sqrt{x}$

$y=0$

$x=3$

$$V = \pi\int_c^d R(y)^2 - r(y)^2\,dy$$

about the line $x=6$



$$V = \pi \int_{y=c}^{d} R(y)^2 - r(y)^2 \, dy$$

$y=0$ (given) and $y=\sqrt{3}$

$R(y) = 6-y^2 \qquad r(y) = 6-3 = 3$

$$V = \pi \int_0^{\sqrt{3}} (6-y^2)^2 \, dy - \pi \int_0^{\sqrt{3}} 3^2 \, dy$$

$$= \pi \int_0^{\sqrt{3}} \left[(36-12y^2+y^4) - 9\right] dy$$

$$= \pi \left(27y - 12y^3/3 + y^5/5\right)_{y=0}^{\sqrt{3}}$$

$$= \pi \left(27\sqrt{3} - 12 * \frac{\cancel{3}\sqrt{3}}{\cancel{3}} + \frac{(\sqrt{3})^5}{5}\right)$$

$$= \pi \left(15\sqrt{3} + \frac{3 * 3\sqrt{3}}{5}\right)$$

$$= (75+9)\frac{\sqrt{3}}{5}\pi = 91.4154$$