7.1 Worksheet #4

Tuesday, October 8, 2019 2:59 PM

$f(x) = -2x^3 + 9x^2 - 10x + 3$

$g(x) = x^2 - 4x + 3$

Last Lecture: Points of intersection.

$$-2x^3 + 9x^2 - 10x + 3 = x^2 - 4x + 3$$
$$-x^2 + 4x - 3 \qquad -x^2 + 4x - 3$$

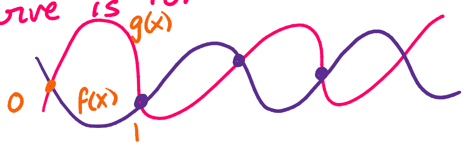
$$-2x^3 + 8x^2 - 6x = 0$$

$$-2x(x^2 - 4x + 3) = 0$$

$$-2x(x-1)(x-3) = 0$$

$x = 0$, $x = 1$, $x = 3$ POINTS OF INTERSECTION

check which curve is "TOP" & "BOTTOM"



Between $0 < x < 1$ $\quad x = 0.5 = 1/2$

$f(x) = -2(1/2)^3 + 9(1/2)^2 - 10/2 + 3 = -1/4 + \frac{9}{4} - 5 = -3$

$g(x) = (1/2)^2 - 4(1/2) + 3$
$= 1/4 - 2 + 3 = 1/4 + 1 = 5/4 = 1.25$

Between 0 & 1, $g(x)$ = top $\quad f(x)$ = Bottom.

Between 1 & 3 ? $\boxed{x = 2}$

$f(2) = -2(2)^3 + 9(2)^2 - 10*2 + 3 = -16 + 36 - 20 + 3$
$= 3$

$g(2) = 4 - 4*4 + 3$
$= 7 - 16 = -9$

Between 1 & 3, $g(x)$ = Bottom & $f(x)$ = Top.

$$\int_0^1 (g(x) - f(x))\,dx + \int_1^3 (f(x) - g(x))\,dx = \text{area bounded by } f(x) \text{ \& } g(x).$$

$$= A_1 + A_2$$

$$A_1 = \int_0^1 (x^2 - 4x + 3) - (-2x^3 + 9x^2 - 10x + 3)\,dx$$

$$= \int_0^1 (\underbrace{x^2 - 9x^2}_{-8x^2} \underbrace{-4x + 10x}_{6x} + \cancel{3} - \cancel{3} + 2x^3)\,dx \quad g(x) - f(x)$$

$$= \boxed{\int_0^1 -8x^2\,dx + \int_0^1 6x\,dx + \int_0^1 2x^3\,dx}$$

$$A1 = -\frac{8}{3} + \frac{\cancel{6}^{3}}{\cancel{2}} + \frac{\cancel{2}^{1}}{\cancel{4}_{2}} = -\frac{8}{3} + 3.5 = \boxed{0.83333}$$

$$A2 = \int_1^3 -2x^3 + 9x^2 - 10x + 3 - (x^2 - 4x + 3)\,dx$$

negative of integrand

$$A_2 = \int_1^3 -2x^3 + 9x^2 - 10x + 3 - (x^2 - 4x + 3)\, dx$$

$$\boxed{= \int_1^3 -2x^3 + 8x^2 - 6x \quad dx}$$

$f(x) - g(x)$

negative of integ

Quick check!

$$= -\cancel{2}\frac{x^4}{\underset{2}{\cancel{4}}}\Bigg|_{x=1}^{3} + 8\frac{x^3}{3}\Bigg|_{x=1}^{3} - \overset{3}{\cancel{6}}\frac{x^2}{\cancel{2}}\Bigg|_{x=1}^{3}$$

$$= \left(-\frac{1}{2}\right)(3^4 - 1) + \frac{8}{3}(3^3 - 1) - 3(3^2 - 1)$$

$$= -\frac{\cancel{80}^{40}}{\cancel{2}} + \frac{8 * 26}{3} - 3 * 8$$

$$= -40 - 24 + \frac{8 * 26}{3} = \boxed{5.3333}$$

area bounded by $f(x)$ & $g(x) = A_1 + A_2 = 0.83333 + 5.33333$

$$= \boxed{6.1666}$$

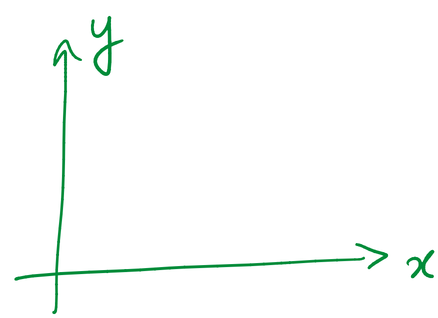
Final Answer

W.S #5 Find area bounded by:

$$x = y^2 - 1 = f(y)$$

and $$x = 1 - y = g(y)$$

$$\int_{y=c}^{d} (\text{RIGHT} - \text{LEFT})\, dy$$



find points of intersection? c, d such that $f(c) = g(c)$ & $f(d) = g(d)$.

$$y^2 - 1 = 1 - y$$

$$-(1-y) \qquad -(1-y)$$

$$y^2 + y - 2 = 0 \longrightarrow \underset{c}{y = -2} \quad \underset{d}{y = 1}$$

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$(y+2)(y-1)$

$$\text{Area} = \int_{y=-2}^{1} ? - ? \; dx$$

$f(y) = y^2 - 1 \qquad\qquad g(y) = 1 - y$

$f(0) = -1 \qquad\qquad g(0) = 1$

$$\text{Area} = \int_{-2}^{1} (1-y) - (y^2 - 1) \, dy$$

$$= \int_{-2}^{1} (1 - y - y^2 + 1) \, dy$$

$$= \int_{-2}^{1} (2 - y - y^2) \, dy$$

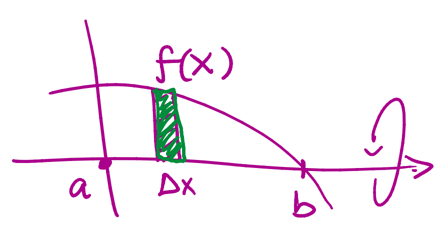
$$= 2y \Big|_{y=-2}^{1} - y^2/2 \Big|_{y=-2}^{1} - y^3/3 \Big|_{y=-2}^{1}$$

$$= 2(\underbrace{1-(-2)}_{3}) - \frac{1}{2}(\underbrace{1-4}_{-3}) - \frac{1}{3}(\underbrace{1-(-8)}_{9})$$

$$\text{Area} = 6 + \frac{3}{2} - 3 = 3 + 1.5 = 4.5$$

7.2: Volume of Solid of Revolution DISK METHOD

$$\text{Volume} = \int_a^b \pi f(x)^2 \, dx$$



Ques: Find the volume of solid obtained by revolving:

$f(x) = \sqrt{\sin x}$ about x-axis

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$f(x) = \sqrt{\sin x}$ about x-axis

between $x = 0$ & $x = 1$.

$$\text{Volume} = \pi \int_{x=0}^{1} \left(\sqrt{\sin x}\right)^2 dx$$

$$= \pi \int_0^1 \sin x \, dx = \pi * -\cos x \Big|_{x=0}^{1}$$

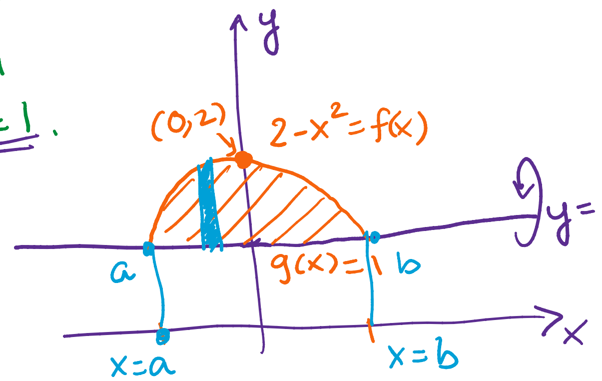
$$= -\pi \cos 1 + \pi \cos 0$$

$$= \pi(1 - \cos 1) = 0.00047$$

example: Find volume of solid formed by revolving region bounded by the graphs of $f(x) = 2 - x^2$

$g(x) = 1$

about the line $y = 1$.



$$\text{Volume} = \int_{x=a}^{b} \pi R(x)^2 dx$$

We need a & b and $R(x)$

$R(x) = 2 - x^2 - 1 = 1 - x^2$

$a = ?$ $b = ?$ Points of intersection for $f(x)$ & $g(x)$

$2 - x^2 = 1 \Rightarrow x^2 = 1$ $x = -1$ or $x = 1$

$$\text{Volume} = \int_{-1}^{1} \pi (1 - x^2)^2 dx = \pi \int_{-1}^{1} (1 - 2x^2 + x^4) dx$$

$$= \pi \int_{-1}^{1} 1 \, dx - 2\pi \int_{-1}^{1} x^2 dx + \pi \int_{-1}^{1} x^4 dx$$

$$= 16\pi/15 \text{ answer!}$$