Tuesday, October 8, 2019 1:43 PM

$(1,1) \quad (4,2) \quad (\underset{=}{\overset{x_2}{9}},3)$ 3 data points → deg 2 poly

$$P_2(x) = 1*\frac{(x-4)(x-9)}{\underset{(x_0-x_1)(x_0-x_2)}{(1-4)(1-9)}} + 2*\frac{(x-1)(x-9)}{\underset{(x_1-x_0)(x_1-x_2)}{(4-1)(4-9)}} + 3*\frac{(x-1)(x-4)}{\underset{(x_2-x_0)(x_2-x_1)}{(9-1)(9-4)}}$$

$$P_2(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

check:

$$P_2(1) = 1*\frac{\cancel{(1-4)}\cancel{(1-9)}}{\cancel{(1-4)}\cancel{(1-9)}} + 0 = 1 \checkmark$$

$$P_2(4) = 0 + 2*\frac{\cancel{(4-1)}\cancel{(4-9)}}{\cancel{(4-1)}\cancel{(4-9)}} = 2 \checkmark$$

$$P_2(9) = 0 + 3*\frac{\cancel{(9-1)}\cancel{(9-4)}}{\cancel{(9-1)}\cancel{(9-4)}} = 3 \checkmark$$

$P_2(x)$ interpolated $(1,1)$ $(4,2)$ & $(9,3)$.

What happens if I add one more data point? $(16,4)$

$(x_0,y_0) \ldots (x_3,y_3)$

$\underset{x_0}{(1,1)}$ $\overset{x_1}{(4,2)}$ $\overset{x_2}{(9,3)}$ & $\overset{x_3}{(16,4)}$

the Lagrange formula needs to redone!

$$P_3(x) = y_0L_0(x) + \overset{2}{y_1}L_1(x) + y_2L_2(x) + \underset{4}{y_3}L_3(x)$$

$$P_3(x) = 1*\frac{(x-4)(x-9)(x-16)}{(1-4)(1-9)(1-16)} +$$

$$2*\frac{(x-1)(x-9)(x-16)}{(4-1)(4-9)(4-16)} +$$

$$3*\frac{(x-1)(x-4)(x-16)}{(9-1)(9-4)(9-16)} +$$

$$4*\frac{(x-1)(x-4)(x-9)}{(16-1)(16-4)(16-9)}$$

Disadvantage of Lagrange: every time a new data point is added, the whole poly. needs to be reconstructed!

Newton's $\underbrace{\text{Divided Difference}}_{??}$ poly. formula overcomes this!

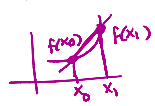
How?

D.D (Divided Difference) of a function $f(x)$

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Newton's Quotient

Slope



Newton's D.D of order 1

$$f[x_i, x_{i+1}] = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

Note: $x_{i+1} = x_i + \Delta x$

$$\boxed{f[x_i, x_{i+1}]} = \boxed{\frac{f(x_i+\Delta x) - f(x_i)}{\Delta x}} \boxed{\simeq f'(x_i)}$$

$$\lim_{\Delta x \to 0} \frac{f(x_i+\Delta x) - f(x_i)}{\Delta x} = f'(x_i)$$

$P_1^N(x)$ = Newton's D.D poly interpolating (x_0, y_0) (x_1, y_1)

$y_0 \downarrow f(x_0)$, $y_1 \downarrow f(x_1)$

$$= f(x_0) + f[x_0, x_1](x - x_0)$$

$$= f(x_0) + \left(\frac{f(x_1) - f(x_0)}{(x_1 - x_0)}\right)(x - x_0)$$

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for $(1,1)$ & $(4,2)$ the Newton's D.D poly is:

$(x_0, f(x_0))$ $(x_1, f(x_1))$

$$P_1^N(x) = 1 + f[1,4](x-1)$$

$$(2-1)\ (x-1)$$

$$p_1^N(x) = 1 + f[1,4](x-1)$$
$$= 1 + \left(\frac{2-1}{4-1}\right)(x-1)$$
$$= 1 + \frac{(x-1)}{3}$$

Check:
$p_1^N(1) = 1$ ✓ $\quad p_1^N(4) = 1 + \frac{(4-1)}{3} = 1+1 = 2$ ✓

Newton's D.D poly for
$\overset{f(x_0)}{(1,1)}$ $\overset{f(x_1)}{(4,2)}$ $\overset{f(x_2)}{(9,3)}$?

3 data points need deg 2 poly.

$$p_2^N(x) = f(x_0) + \underbrace{f[x_0,x_1]}_{1/3}(x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

D.D of order 2 w.r.t $f(x)$

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2] - f[x_0,x_1]}{x_2 - x_0}$$

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	
x_0 $\boxed{1}$	1	$f[x_0,x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{1}{3}$	
$\boxed{4}$	$\boxed{2}$		$\frac{1/5 - 1/3}{9-1} = \frac{f[x_1,x_2] - f[x_0,x_1]}{x_2-x_0}$, $\frac{-2}{8*15} = \frac{-1}{60}$
x_2 9	$\boxed{3}$	$\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{1}{5}$, $f[x_1,x_2]$	

$$p_2^N(x) = 1 + \frac{(x-1)}{3} + \left(-\frac{1}{60}\right)(x-1)(x-4)$$

check: $p_2^N(1) = 1$, $p_2^N(4) = 2$ & $p_2^N(9) = 3$.

Construct the Newton's D.D poly interpolating
$(1,1)$, $(4,2)$, $(9,3)$ & $(16,4)$.

4 data points $\Rightarrow$ use $p_3^N(x)$

$$p_3^N(x) = f(x_0) + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$
$$+ f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$$

Newton's D.D of order 3 !

x_i	$f(x_i)$	$f[x_i,x_{i+1}]$	$f[x_i,x_{i+1},x_{i+2}]$	$f[x_0,x_1,x_2$
$\boxed{1}$	1	$1/3$	$\frac{1/5 - 1/3}{9-1} = -1/60$	
$\boxed{4}$	2	$\boxed{1/5}$		$\frac{-1/210 +}{x_3 - \ }$, $16 -$
$\boxed{9}$	3	$\boxed{1/7}$	$\frac{1/7 - 1/5}{16-4} = \frac{-2}{\cancel{12}_{6}*35} = -\frac{1}{210}$	
$\boxed{16}$	4			

$\frac{-60+}{15}$

$, x_3]$

$\frac{1/60}{x_0} = \boxed{0.007365}$

$\frac{210}{5 * 210 * 60} = \frac{\cancel{150}^{10}}{\cancel{15} * 210 * 6\cancel{0}}$

$= \frac{1}{}$

$$\boxed{p_3^N(x) = 1 + \frac{(x-1)}{3} - \frac{(x-1)(x-4)}{60} + \frac{1}{1260}(x-1)(x-4)}$$

$$= 1 + (x-1)*\Big(1/3 + (x-4)\Big(-\frac{1}{60} + \frac{(}{}$$

Horner's Scheme (Neste

<u>Disadvantage of Newton's D.D poly</u>:

Error formula is in an impractical

<u>Work Around</u>: poly interpolates n+1 distinct

is always unique!

$$p_n^L(x) = p_n^N(x)$$

BOT

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dis

for interpolating $(x_0, y_0), \ldots (x_n, y_n)$ p

so we use the "nice" Lag poly error f

formula instead!

5 * 210 * 60

$$\boxed{(x-9)} = \frac{1}{12600}$$

$$\left(\frac{x-9)}{1260}\right)$$

d Approach)

form !

points

H LAGRANGE &
WTON POLY GIVE
THE SAME CURVE
stinct
oints !

representation