Thursday, October 10, 2019 12:21 PM

1) $f(x) = e^x$ $\quad x_0 = 0 \quad x_1 = 0.5 \quad x_2 = 1$ & $x_3 = -1$

$$\left| f(x) - p_3(x) \right| = \left| \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(n+1)!} f^{(n+1)}(c_x) \right|$$

$n=3$

c_x lies between -1 & 1.

Minimum of all x_is $= -1$

Maximum of all x_is $= 1$

$n=3 \quad n+1 = 4 \quad (n+1)! = 24$

$$|f(x) - p_3(x)| = \frac{|(x-0)(x-0.5)(x-1)(x-(-1))|}{4!} e^{c_x}$$

$$= \frac{|x(x-0.5)(x-1)(x+1)|}{24} e^{c_x} \qquad -1 \le x \le 1$$

$-1 \le c_x \le 1$

$\downarrow$ $c_x = 1$

$$\le \frac{|x(x-0.5)(x-1)(x+1)|}{24} e^1 \qquad -1 \le x \le 1$$

Recall in Taylor poly error

$$\left| \frac{(x-a)^{n+1}}{(n+1)!} \right| \left| f^{(n+1)}(c_x) \right|$$

$a=0$

$$\frac{|x|^{n+1}}{(n+1)!} \left| f^{(n+1)}(c_x) \right|$$

$x=1$

$-1 \le x \le 1$

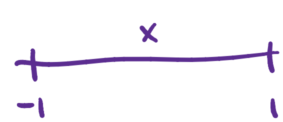
$$\frac{1^{n+1}}{(n+1)!} \left| f^{(n+1)}(c_x) \right|$$

How to bound

$$|x(x-0.5)(x+1)(x-1)|$$

$$= |x| \, |(x-0.5)(x+1)(x-1)|$$

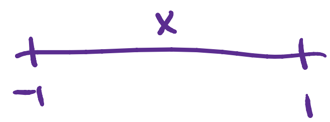
$\downarrow$ use $x=1$



use $x=1$

$\leq 1 \quad |(x-0.5)(x+1)(x-1)|$

$= \quad |x-0.5| \; |(x+1)(x-1)|$

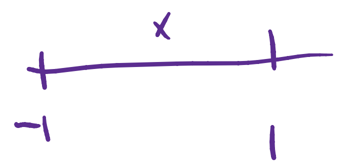


use $x=-1$

$\leq \quad |-1.5| \quad |(x+1)(x-1)|$

$= \quad 1.5 \quad |(x+1)(x-1)|$

$= \quad 1.5 \quad |x+1| \; |x-1|$



$x=1$ $\qquad$ $x=-1$

$= \quad 1.5 * 2 * 2$

$= \quad 6$

Alternate Soln: We want a bound for

$g(x) = x(x-0.5)(x-1)(x+1)$

and use Calculus to find maximum value.

$g'(x) = 0 \Rightarrow x = x^*$

check sign $g''(x^*)$ for max value

Final Bound

$$|f(x) - p_3(x)| \leq \frac{|x(x-0.5)(x-1)(x+1)|}{24} e$$

$$\leq \frac{6}{24} e = \frac{e}{4} = 0.67957$$

for $-1 \leq x \leq 1$.

Splines: Example

#11 (Exercises 4.3)

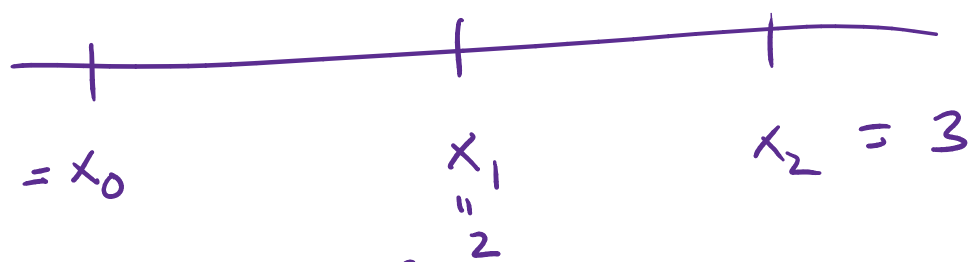
Splines : Ex...

#11 (Exercises 4.3)

$$S(x) = \begin{cases} -5 + 8x - 6x^2 + 2x^3 = S_1(x) & 1 \le x \le 2, \\ 27 - 40x + 18x^2 - 2x^3 = S_2(x) & 2 \le x \le 3. \end{cases}$$

Verify that $S(x)$ is a cubic spline on $[1,3]$.
Is it a natural cubic spline?

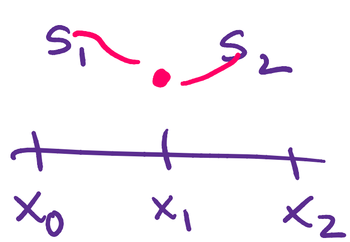
$S(x)$ is a piece wise cubic poly on $[1,3]$



Properties to be satisfied:

① $S(x)$ is a cubic poly on $[x_0, x_1]$ and $[x_1, x_2]$.
Satisfied by definition of $S(x)$ ✓

② $S(x_i) = y_i$ No interpolating info given to us so this condition is not applicable to our problem.

③ 

$x_1 \rightarrow$ trying to glue S_1 & S_2

$$\lim_{x \to x_1^-} s'(x) = \lim_{x \to x_1^+} s'(x)$$

that is

$$\lim_{x \to 2^-} s'(x) = \lim_{x \to 2^+} s'(x)$$

$$\lim_{x \to 2} s_1'(x) \overset{?}{=} \lim_{x \to 2} s_2'(x)$$

$$s_1(x) = -5 + 8x - 6x^2 + 2x^3$$

$$s_1'(x) = 0 + 8 - 12x + 6x^2$$

$$\lim_{x \to 2} s_1'(x) = 8 - 24 + 24 = 8$$

$$s_2(x) = 27 - 40x + 18x^2 - 2x^3$$

$$s_2'(x) = -40 + 36x - 6x^2$$

$$s_2'(2) = -40 + 72 - 24$$

$$= 8$$