Tuesday, October 8, 2019 12:32 PM

math 4329

$(1,1)$, $(4,2)$, $(9,3)$ and $(16,4)$

$$P_3(x) = \frac{(x-4)(x-9)(x-16)}{(1-4)(1-9)(1-16)} + 2\frac{(x-1)(x-9)(x-16)}{(4-1)(4-9)(4-16)}$$

$$+ 3\frac{(x-1)(x-4)(x-16)}{(9-1)(9-4)(9-16)} + 4\frac{(x-1)(x-4)(x-9)}{(16-1)(16-4)(16-9)}$$

Lagrange formula for interpolating 4 data points.

Disadvantage: when new data points get added.

Newton's Divided Difference polynomial.

What is Divided Difference?

Newton's Quotient: $\lim_{\Delta x \to 0} \underbrace{\frac{\overbrace{f(\underset{x_1}{x+\Delta x}) - f(\underset{x_0}{x})}^{\text{Newton's Quotient}}}{\Delta x}}_{\text{Divided Difference}}$

$f[x, x+\Delta x]$

$f[x_0, x_1]$ = Divided Difference (D.D) of $f(x)$ with resp. to x_0, x_1 (FIRST ORDER D.D)

$$= \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Given $\underset{}{\overset{x_0\ y_0}{(1,1)}}$ and $\overset{x_1\ y_1}{(4,2)}$ construct a Newton's D.D poly. interpolating the above 2 points.

$P_1^N(x)$ = Newton's D.D poly

Reminds us of: $\left(f(x_0) + f'(x_0)(x-x_0)\right)$

$$P_1^N(x) = \underset{y_0}{f(x_0)} + \underset{(y_1-y_0)/(x_1-x_0)}{f[x_0,x_1]}(x-x_0)$$

$$= 1 + \frac{(2-1)}{(4-1)}(x-1)$$

$$P_1^N(x) = 1 + \frac{1}{3}(x-1)$$

check if Newton's D.D poly interpolates $(1,1)$ and $(4,2)$

$P_1^N(1) = 1 + \frac{1}{3}*0 = 1$ ✓

$P_1^N(4) = 1 + \frac{1}{3}(4-1) = 2$ ✓

$P_1^N(x)$ interpolates $(1,1)$ and $(4,2)$

Suppose I want to construct poly using Newton D.D formula interpolating $\underset{x_0\ y_0}{(1,1)}, \underset{x_1\ y_1}{(4,2)}$ and $\underset{x_2\ y_2}{(9,3)}$. $n=2$, $n+1$ points

$$P_2^N(x) = \underset{y_0}{f(x_0)} + \underset{\substack{\downarrow\\ \text{Newton's D.D}\\ \text{of order 1}}}{f[x_0,x_1]}(x-x_0) + \underset{\substack{\downarrow\\ \text{Newton's}\\ \text{D.D of}\\ \text{order 2}}}{f[x_0,x_1,x_2]}(x-x_0)(x-x_1)$$

x_i	y_i = $\boxed{f(x_i)}$	$f[x_i, x_{i+1}]$	$f[x_0,x_1,x_2]$
x_0 1	1	$\frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{2-1}{4-1} = 1/3$	
4	2		$= \frac{1/5 - 1/3}{9-1} = \frac{3-5}{8*15} = \frac{-2}{8*15} = -1/60$
x_2 9	3	$f[x_1,x_2] = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{3-2}{9-4} = 1/5$	

$$P_2^N(x) = 1 + \frac{(x-1)}{3} + \left(-\frac{1}{60}\right)(x-1)(x-4) = P_1^N(x) - \frac{1}{60}(x-1)(x-4)$$

check:

$P_2^N(1) = 1 - 0 = 1$ ✓

$P_2^N(4) = 2 - 0 = 2$ ✓

$P_2^N(9) = 1 + \frac{9-1}{3} - \frac{1}{60}(9-1)(9-4)$

$= 3$ (verify!)

$p_2^N(1) = 1 - 0 = \ldots$

$p_2^N(4) = 2 - 0 = 2$ ✓

$= 3$ (verify!)

$(1,1)$, $(4,2)$, $(9,3)$, $(16,4)$ — New Data Point. $n=3$, $n+1=4$ data points

($x_0\ y_0$, $x_1\ y_1$, $x_2\ y_2$, $x_3\ y_3$)

Construct
$$\boxed{\begin{aligned} P_3^N(x) = {} & f(x_0) + f[x_0,x_1](x-x_0) \\ & + f[x_0,x_1,x_2](x-x_0)(x-x_1) \\ & + f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2) \end{aligned}}$$

Note: $p_3^N(x) = p_2^N(x) + \underbrace{f[x_0,x_1,x_2,x_3]}_{???}(x-x_0)(x-x_1)(x-x_2)$

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$$

x_i	y_i $f(x_i)$	$f[x_i,x_{i+1}]$	$f[x_i,x_{i+1},x_{i+2}]$	$f[x_0,x_1,x_2,x_3]$
1	1	$1/3$	$\frac{1/5-1/3}{9-1} = -1/60$	$\frac{-1/210-(-1/60)}{16-1} = \boxed{-1/700}$
$\boxed{4}$	2	$(3-2)/9-4 = \boxed{1/5}$	$\frac{1/7-1/5}{16-4} = \frac{-2}{35} * \frac{1}{12} = \frac{-1}{35*6} = \frac{-1}{210}$	
9	3	$1/16-9 = \boxed{1/7}$		
$\boxed{16}$	4			

$$p_3^N(x) = 1 + \frac{(x-1)}{3} - \frac{(x-1)(x-4)}{60} - \frac{1}{700}(x-1)(x-4)(x-9)$$

Disadvantage of Newton's D.D: error representation formula is not a very useful form!

for $n+1$ distinct pts

$p_n^{Lag}(x) = p_n^N(x)$ so I can take advantage of nice error formula for Lagrange poly.