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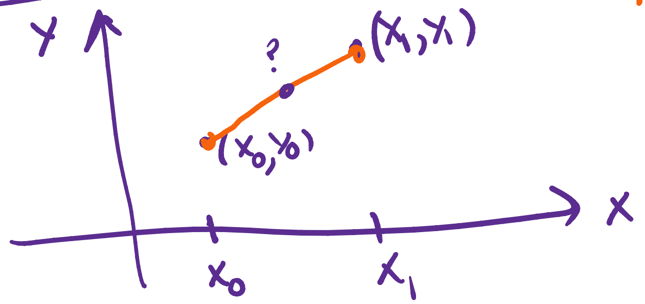
Monday, September 30, 2019 3:18 PM

Interpolation:

simplest → straight line



Given $(1, 2)$, $(3, 5)$ construct a polynomial curve interpolating the given points.

$\cdot (3, 5)$
$(x_1\ y_1)$

$\cdot (1, 2)$
$(x_0\ y_0)$

$$\frac{y - y_1}{x - x_1} = m = \frac{y_0 - y_1}{x_0 - x_1}$$

$$m = \frac{2-5}{1-3} = \frac{-3}{-2} = 3/2 \longrightarrow \frac{y_1 - y_0}{x_1 - x_0} = \frac{5-2}{3-1} = 3/2$$

$x_1 = 3$
$y_1 = 5$

$$\frac{y-5}{x-3} = 3/2 \Rightarrow y = 5 + 3/2\,(x - 3)$$

$\boxed{P_1(x) = 5 + 3/2\,(x-3)}$ interpolates $(1,2)$ and $(3,5)$

check: $P_1(1) = 5 + 3/2\,(1-3) = 5 + \frac{3}{2} * (-2) = 2$

$P_1(3) = 5 + 3/2\,(3-3) = 5$

(x_0, y_0) and (x_1, y_1)

$$P_1(x) = y_0 \frac{(x_1 - x)}{(x_1 - x_0)} + y_1 \frac{(x - x_0)}{(x_1 - x_0)}$$

{ scaling factor }

Check:

$$\boxed{\begin{aligned} P_1(x_0) &= y_0 \frac{(x_1 - x_0)}{(x_1 - x_0)} + 0 = y_0 \\ P_1(x_1) &= 0 + y_1 \frac{(x_1 - x_0)}{x_1 - x_0} = y_1 \end{aligned}}$$

$P_1(x_1) = \dfrac{\quad}{x_1 - x_0}$

$$\boxed{p_1(x) = y_0\, l_0(x) + y_1\, l_1(x)}$$ (straight line obtained earlier)

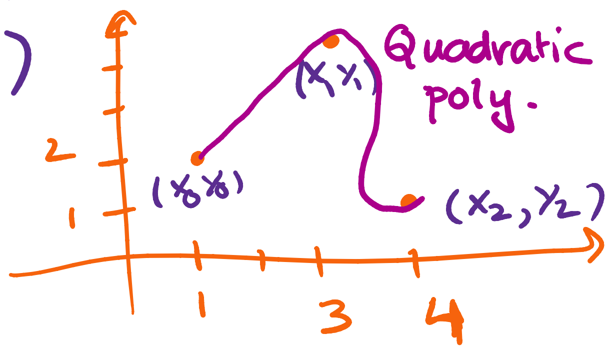
where $l_0(x) = \dfrac{x_1 - x}{x_1 - x_0}$ and $l_1(x) = \dfrac{x - x_0}{x_1 - x_0}$.

verify → $p_1(1) = 2$ $\quad P_1(3) = 5$

$l_0(x) = \dfrac{3-x}{3-1}$ $\quad l_1(x) = \dfrac{x-1}{3-1}$

$$\boxed{\begin{array}{lll} x_0 = 1 & x_1 = 3 & x_2 = 4 \\ y_0 = 2 & y_1 = 5 & y_2 = 1 \end{array}}$$

Construct an interpolating poly passing through $(1,2)$, $(3,5)$ & $(4,1)$



Quadratic poly.

How to derive such a formula:

$p(x) = ax^2 + bx + c$

3 data points give me 3 conditions to derive a, b & c

Condn 1: $p(1) = 2$

$a * 1^2 + b * 1 + c = 2 \rightarrow \boxed{a + b + c = 2}$

Condn 2: $p(3) = 5$

$a * 3^2 + b * 3 + c = 5 \rightarrow \boxed{9a + 3b + c = 5}$

Condn 3: $p(4) = 1$

$\boxed{16a + 4b + c = 1}$

3 equations & 3 unknowns a, b & c.

solve this → $\begin{bmatrix} 1 & 1 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$

solve this $\rightarrow$ $\begin{bmatrix} \vdots & & \\ 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} \vdots \\ c \end{bmatrix} \begin{bmatrix} \vdots \\ 1 \end{bmatrix}$

This method is reliable but tedious!

example 4 data points $\rightarrow$ solve 4×4 system

Our Approach: (x_0, y_0) (x_1, y_1) & (x_2, y_2)

$$p_2(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$$L_0(x) = \begin{cases} 1 & \text{if } x = x_0 \\ 0 & \text{otherwise} \end{cases} \qquad L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}.$$

$$L_1(x) = \begin{cases} 1 & x = x_1 \\ 0 & \text{otherwise} \end{cases}$$

$$L_2(x) = \begin{cases} 1 & x = x_2 \\ 0 & \text{otherwise} \end{cases}$$

$p_2(x_0) = y_0$, $p_2(x_1) = y_1$ and $p_2(x_2) = y_2$.