Math4329

Thursday, April 9, 2020 9:44 AM

Correction to 15/Page158

$$S(x) = \begin{cases} S_1(x) = x^3 + 2x^2 + 1 & 1 \le x \le 2 \\ S_2(x) = -2x^3 + \beta x^2 - 36x + 25 & 2 < x \le 3 \end{cases}$$

2 = Gluing point

What value of β makes $S(x)$ a spline?

find β s.t. $S(x)$, $S'(x)$ & $S''(x)$ are cts on $[1,3]$.

$S(x)$ is cts for $1 < x < 2$ and $2 < x < 3$ but we need to check at $x = 2$ (This was my mistake in the last lecture)

$S_1(x) = x^3 + 2x^2 + 1$ $\boxed{x = 2}$? $S_2(x) = -2x^3 + \beta x^2 - 36x + 25$

$S_1'(x) = 3x^2 + 4x$ $S_2'(x) = -6x^2 + 2\beta x - 36$

$S_1''(x) = 6x + 4$ $S_2''(x) = -12x + 2\beta$

Using $S_1''(2) = S_2''(2)$ we find β:

$S_1''(2) = (6*2) + 4 = 16$ $S_2''(2) = -24 + 2\beta$

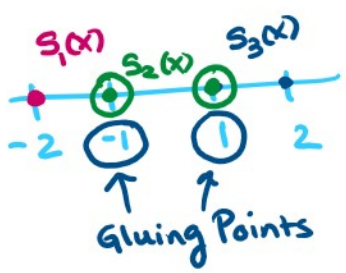
$S_2''(2) = S_1''(2) \Rightarrow -24 + 2\beta = 16 \Rightarrow 2\beta = 40 \Rightarrow \boxed{\beta = 20}$

Check if
$$S(x) = \begin{cases} x^3 + 2x^2 + 1 & 1 \le x \le 2 \\ -2x^3 + 20x^2 - 36x + 25 & 2 < x \le 3 \end{cases}$$

β value

Ques:

$$S(x) = \begin{cases} S_1(x) = (x+1)^3 & -2 \le x \le -1, \\ S_2(x) = ax^3 + bx^2 + cx + d & -1 < x \le 1, \\ S_3(x) = (x-1)^2 & 1 < x \le 2. \end{cases}$$



find a, b, c & d so that $S(x)$ is a spline.

... $S(x)$, $S'(x)$ & $S''(x)$ are cts on

Gluing Points spline.

find a,b,c,d so that $s(x)$, $s'(x)$ & $s''(x)$ are cts on $[-2,2]$.

Idea: use cty of $s(x)$, $s'(x)$ and $s''(x)$ to obtain a System of equations in unknowns a,b,c,d. Solve for a,b,c,d.

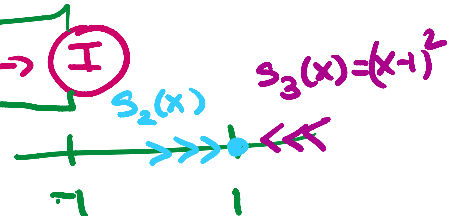
Tip*: Easiest to solve $\left.\begin{array}{l} s_2''(-1)=s_1''(-1) \\ s_2''(1)=s_3''(1) \end{array}\right\}$

① cty of $s(x)$ gives: $\lim\limits_{x\to -1^+} s(x) = \lim\limits_{x\to -1^-} s(x)$

$$\lim_{x\to -1} s_2(x) = \lim_{x\to -1} s_1(x) = s_1(-1) = (-1+1)^3 = 0$$

$$a(-1)^3 + b(-1)^2 + c(-1) + d = 0$$

$$\boxed{-a + b - c + d = 0} \to \text{I}$$



① cty of $s(x)$ at $x=1$

$$\lim_{x\to 1^-} s(x) = \lim_{x\to 1^+} s(x)$$

$s_2(x) = ax^3+bx^2+cx+d$

$$\lim_{x\to 1} s_2(x) = \lim_{x\to 1} s_3(x) = s_3(1)$$

$$\boxed{a+b+c+d = 0} \to \text{II}$$

② cty of $s'(x)$ at $x=-1$ and cty at $x=1$.

$$\lim_{x\to -1^+} s'(x) = \lim_{x\to -1^-} s'(x) \qquad \lim_{x\to 1^-} s'(x) = \lim_{x\to 1^+} s'(x)$$

$$\lim_{x\to -1} s_2'(x) = \lim_{x\to -1} s_1'(x)$$

$s_2(x) = ax^3+bx^2+cx+d$

$s_2'(x) = 3ax^2+2bx+c$

Note: d gets killed

$s_2'(-1) = 3a-2b+c$

$s_2'(-1) = s_1'(-1) \to s_1'(x) = ?$

$s_1(x) = (x+1)^3$

2

$s_2'(-1) = s_1'(-1)$

$s_1(x) = (x+1)^3$
$s_1'(x) = 3(x+1)^2$
$s_1'(-1) = 0$

$s_2'(-1) = s_1'(-1) \Rightarrow$ $\boxed{3a - 2b + c = 0}$ III

$s_2'(1) = s_3'(1) \Rightarrow$

$s_2'(x) = 3ax^2 + 2bx + c$
$s_3'(x) = 2(x-1) \to s_3'(1) = 0$

$s_2'(1) = s_3'(1) \to$ $\boxed{3a + 2b + c = 0}$ IV

Get 2 more equations using:

$\boxed{s_2''(-1) = s_1''(-1)}$ and $\boxed{s_2''(1) = s_3''(1)}$

where

$s_2'(x) = 3ax^2 + 2bx + c$ $\quad s_1'(x) = 3(x+1)^2$

$s_2''(x) = 6ax + 2b$ $\quad s_1''(x) = 6(x+1)$

vanished from $s_2''(x)$ $\quad s_1''(-1) = 0$

$s_3'(x) = 2(x-1)$

$\boxed{s_3''(x) = 2}$

$s_2''(-1) = s_1''(-1) = 0$

$\boxed{-6a + 2b = 0} \to$ V

$s_2''(1) = s_3''(1) = 2$

$\boxed{6a + 2b = 2} \to$ VI

Recall the TIP: use $s_2''(-1) = s_1''(-1)$ and $s_2''(1) = s_3''(1)$ to obtain a & b then use $s_2'(-1) = s_1'(-1)$ and $s_2'(1) = s_3'(1)$ to get c.

Finally use $s_2(1) = s_3(1)$ to get d.

to get c.

finally use $S_2(1) = S_3(1)$ to get d.

use V & VI $\quad -6a+2b=0$ to obtain a&b.

$$\begin{array}{r} -6a+2b=0 \\ +\quad 6a+2b=2 \\ \hline \end{array}$$

$$-6a+2b+6a+2b = 0+2$$
$$4b = 2 \text{ or } \boxed{b=1/2=0.5}$$

use $b=0.5$ to get a:

$$-6a+2b=0 \rightarrow -6a+2*0.5=0$$
$$-6a+1=0$$
$$\boxed{a=1/6}$$

Solve for c using $a=1/6$ and $b=1/2$ in IV:

(IV) $\quad 3a+2b+c=0$

$$3\left(\tfrac{1}{6}\right)+2\left(\tfrac{1}{2}\right)+c=0$$
$$\tfrac{1}{2}+\tfrac{2}{2}+c=0 \Rightarrow \boxed{c=-3/2}$$

To obtain d, I use II and $a=1/6$, $b=1/2$, $c=-3/2$

(II) $\quad a+b+c+d=0$

$$1/6+1/2+-\tfrac{3}{2}+d=0$$
$$1/6-1+d=0 \Rightarrow \boxed{d=5/6}$$

$$S_2(x) = ax^3+bx^2+cx+d$$
$$= 1/6x^3+1/2x^2-3/2x+5/6 \quad \text{Answer}$$

In WS 8b, #1

$$S(x) = \begin{cases} S_1(x) = x^3 & 0 \le x \le \boxed{2} \\ S_2(x) = -0.5(x-1)^3+a(x-1)^2+b(x-1)+c & \boxed{2} < x \le 3 \end{cases}$$

2 → Gluing point

$S_1(2) = S_2(2) \rightarrow$ 1

$2 \rightarrow$ Gluing point

use $s_2(2) = s_1(2) \rightarrow ①$

$s_2'(2) = s_1'(2) \rightarrow ②$

$s_2''(2) = s_1''(2) \rightarrow ③$

3 eq^{n}s 3 unknowns a, b, c

Afua's Ques:

$$S(x) = \begin{cases} s_1(x) = ax^3 & -1 \le x \le 0 \\ s_2(x) = 3x^3 + bx^2 & 0 < x < \boxed{1} \\ s_3(x) = cx & \boxed{1} \le x \le 2 \end{cases}$$

$s_1(0) = s_2(0)$ $\quad$ $s_2(1) = s_3(1)$

$s_1'(0) = s_2'(0)$ $\quad$ $s_2'(1) = s_3'(1)$

$s_1''(0) = s_2''(0)$ $\quad$ $s_2''(1) = s_3''(1)$

chapter 5 : Numerical Integration & Diff.

Numerical Derivatives

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \longrightarrow \text{Newton Quotient}$$

Num. Derivative

Forward Difference $\quad D_h^+ f(x) = \dfrac{f(x+h) - f(x)}{h}, \; h > 0$

Backward Difference $\quad D_h^- f(x) = \dfrac{f(x) - f(x-h)}{h}, \; h > 0$

Newton D.D

Given

x	$f(x)$	$f[x_i, x_{i+1}]$
-1	-0.45	
0	0	$\rightarrow \dfrac{0-(-0.45)}{0-(-1)} = 0.45$
1	0.5	

1	0.5

$D_1^+ f(-1) = D_h^+ f(x) \quad x=-1 \quad h=1$

$= \left(f(-1+h) - \underbrace{f(-1)}_{=0.45}\right)/h = \dfrac{\overset{0}{f(0)} - \overset{-0.45}{f(-1)}}{1} = \dfrac{0+0.45}{1}$

$D_1^+ f(-1) = f[-1,0]$ Newton D.D with $x_0 = -1,\ x_1 = 0$.

x	$f(x) = \cos(\pi x)$
0.5	0
1	-1
2	1

$f(0.5) = \cos(\pi/2) = 0$

$f(1) = \cos(\pi) = -1$

$f(2) = \cos(2\pi) = 1$